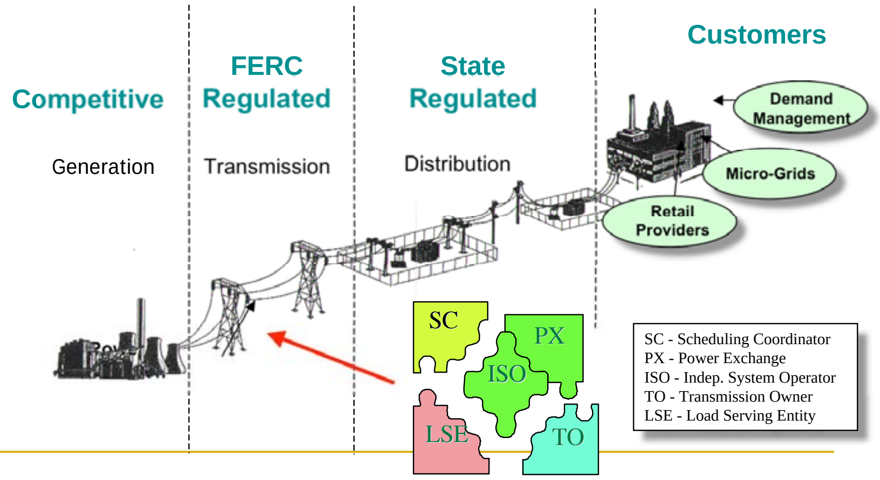
Hedging Fixed Price Load Following Obligations in a Competitive Wholesale Electricity Market

Shmuel Oren
oren@ieor.berkeley.edu
IEOR Dept. , University of California at Berkeley
and Power Systems Engineering Research Center (PSerc)
(Based on joint work with Yumi Oum and Shijie Deng)

Centre for Energy and Environmental Markets
University of New South Wales
October 29, 2008

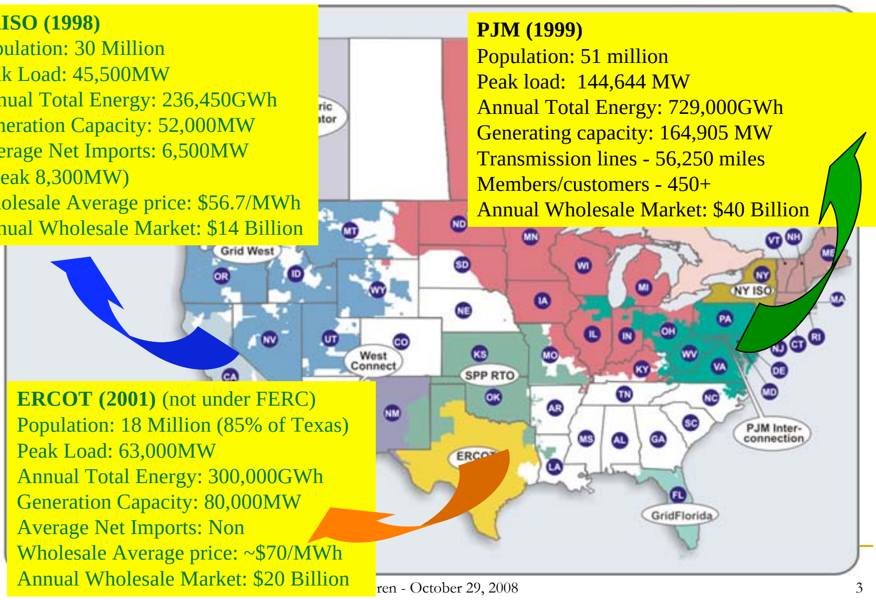
POWER INDUSTRY RESTRUCTURING



Scale of Regional Transmission Organizations
(Cover half the states and 70% of load)

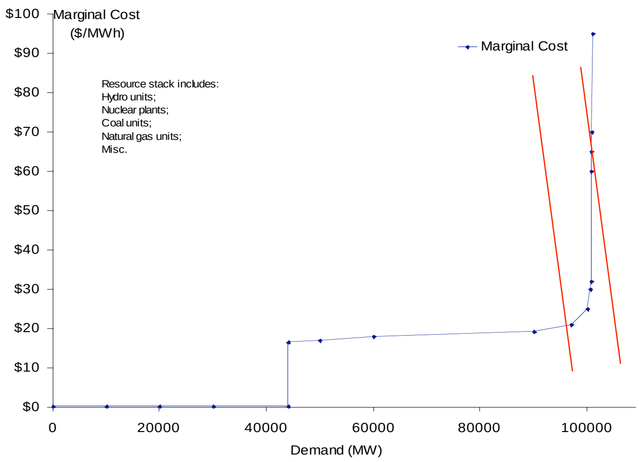
CAISO (1998)
Population: 30 Million
Peak Load: 45,500MW
Annual Total Energy: 236,450GWh
Generation Capacity: 52,000MW
Average Net Imports: 6,500MW
(Peak 8,300MW)
Wholesale Average price: $56.7/MWh
Annual Wholesale Market: $14 Billion

PJM (1999)
Population: 51 million
Peak load: 144,644 MW
Annual Total Energy: 729,000GWh
Generating capacity: 164,905 MW
Transmission lines - 56,250 miles
Members/customers - 450+
Annual Wholesale Market: $40 Billion



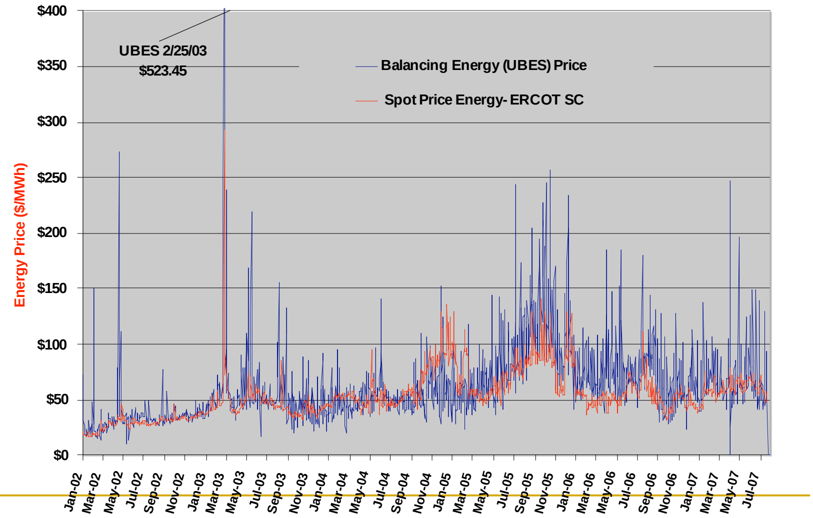
ERCOT (2001) (not under FERC)
Population: 18 Million (85% of Texas)
Peak Load: 63,000MW
Annual Total Energy: 300,000GWh
Generation Capacity: 80,000MW
Average Net Imports: Non
Wholesale Average price: ~$70/MWh
Annual Wholesale Market: $20 Billion

Typical Electricity Supply and Demand Functions

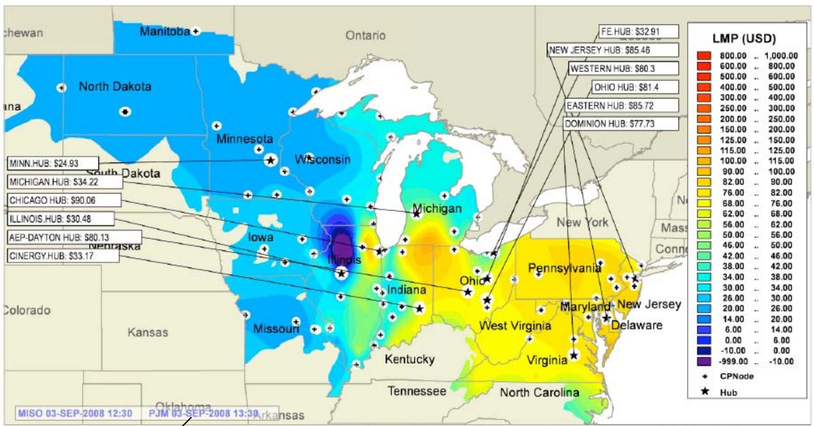


ERCOT Energy Price – On peak Balancing Market vs. Seller's Choice

January 2002 thru July 2007



Locational Real Time Marginal Prices at PJM and MISO



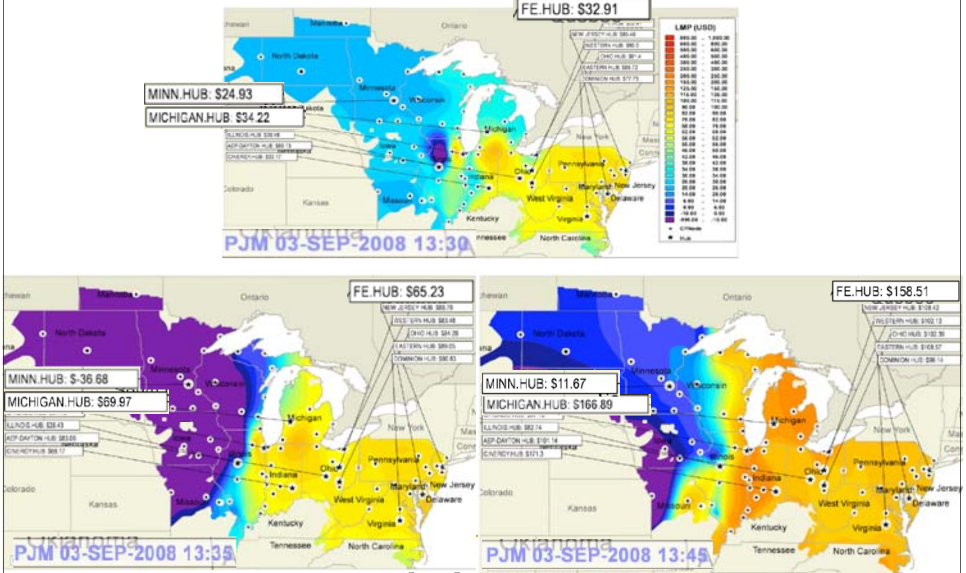
This image will be refreshed in 3 Minutes, 4 Seconds. Please hit crtl-F5 to manually refresh this page.

Midwest ISO Market data is based on Eastern Standard Time (EST) while PJM Market data is based on Eastern Prevailing Time.

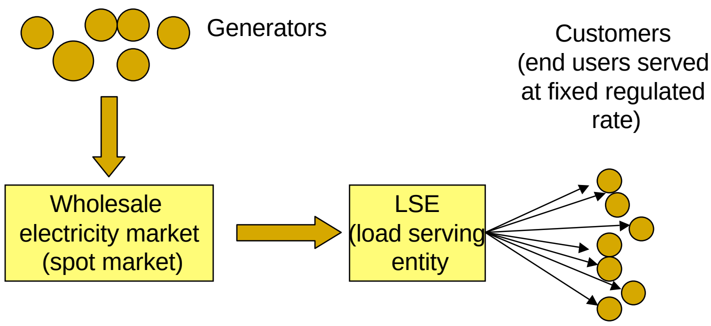
PJM 03-SEP-2008 13:30

Midwest ISO We manage power. pjm

Dispatch is reoptimized every five minutes and LMP updated to reflect shadow prices on transmission constraints

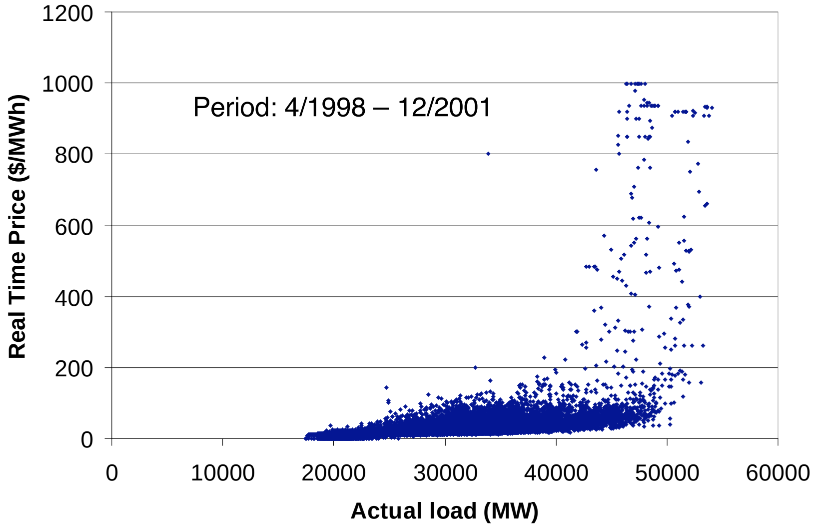


Electricity Supply Chain

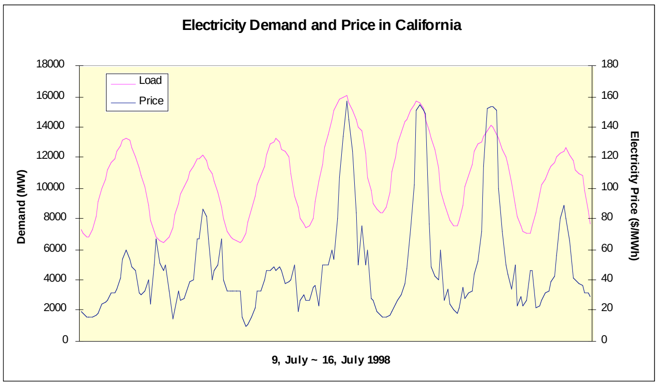


Similar exposure is faced by a trader with a fixed price load following obligation (such contracts were auctioned off in New Jersey and Montana to cover default service needs)

Both Price and Quantity are Volatile (PJM Market Price-Load Pattern)



Price and Demand Correlation



Correlation coefficients:

0.539 for hourly price and load from 4/1998 to 3/2000 at Cal PX

0.7, 0.58, 0.53 for normalized average weekday price and load in Spain, Britain, and Scandinavia, respectively

Volumetric Risk for Load Following Obligation

- Properties of electricity demand (load)
  - Uncertain and unpredictable
  - Weather-driven → volatile
- Sources of exposure
  - Highly volatile wholesale spot price
  - Flat (regulated or contracted) retail rates & limited demand response
  - Electricity is non-storable (no inventory)
  - Electricity demand has to be served (no "busy signal")
  - Adversely correlated wholesale price and load

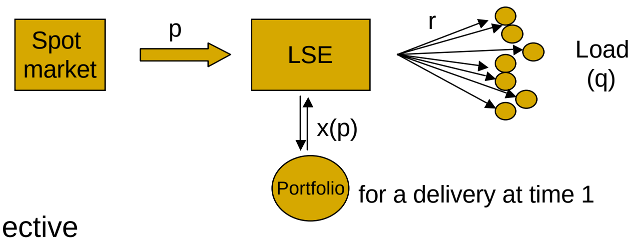
Covering expected load with forward contracts will result in a contract deficit when prices are high and contract excess when prices are low resulting in a net revenue exposure due to load fluctuations

Tools for Volumetric Risk Management

- Electricity derivatives
  - Forward or futures
  - Plain-Vanilla options (puts and calls)
  - Swing options (options with flexible exercise rate)
- Temperature-based weather derivatives
  - Heating Degree Days (HDD), Cooling Degree Days (CDD)
- Power-weather Cross Commodity derivatives
  - Payouts when two conditions are met (e.g. both high temperature & high spot price)
- Demand response Programs
  - Interruptible Service Contracts
  - Real Time Pricing

Volumetric Static Hedging Model Setup

- One-period model
  - At time 0: construct a portfolio with payoff x(p)
  - At time 1: hedged profit Y(p,q,x(p)) = (r-p)q+x(p)



- Objective
  - Find a zero cost portfolio with exotic payoff which maximizes expected utility of hedged profit under no credit restrictions.

Mathematical Formulation

- Objective function

Utility function over profit

$$\max_{x(p)} E\left[U[(r-p)q + x(p)]\right]$$

Joint distribution of p and q

$$\equiv \max_{x(p)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U[(r-p)q + x(p)]\, f(p,q)\, dq\, dp$$

- Constraint: zero-cost constraint

$$\frac{1}{B} E^{Q}[x(p)] = 0$$

! A contract is priced as an expected discounted payoff under risk-neutral measure

Q: risk-neutral probability measure

B : price of a bond paying $1 at time 1

Optimality Condition

The Lagrange multiplier is determined so that the constraint is satisfied

$$E\left[U'\left((r-p)q+x^*(p)\right)\mid p\right]=\lambda^*\frac{g(p)}{f_p(p)}$$

- Mean-variance utility function:

$$E[U(Y)]=E[Y]-\frac{1}{2}aVar(Y)$$

$$x^*(p)=\frac{1}{a}\left(1-\frac{\frac{g(p)}{f_p(p)}}{E^Q\left[\frac{g(p)}{f_p(p)}\right]}\right)+E^Q[E[y(p,q)\mid p]]\frac{\frac{g(p)}{f_p(p)}}{E^Q\left[\frac{g(p)}{f_p(p)}\right]}-E[y(p,q)\mid p]$$

Illustrations of Optimal Exotic Payoffs Under Mean-Var Criterion

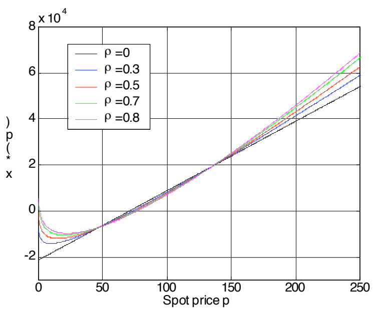
Bivariate lognormal distribution:

$(\log p, \log q) \sim N(4,\ 0.7^2,\ 5.69,\ 0.2^2,\ \rho)$ under P & Q

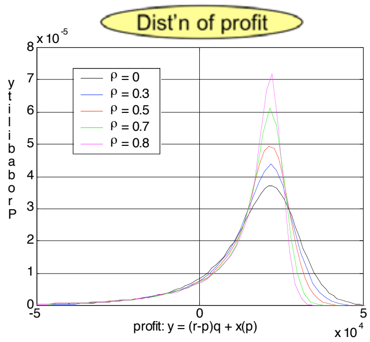
$\begin{pmatrix} E[p]=\$70/MWh,\ \sigma(p)=\$56/MWh \\ E[q]=300MWh,\ \sigma(q)=60MWh \end{pmatrix}$

$r=\$120/MWh$ (flat retail rate)

Optimal exotic payoff



Dist'n of profit

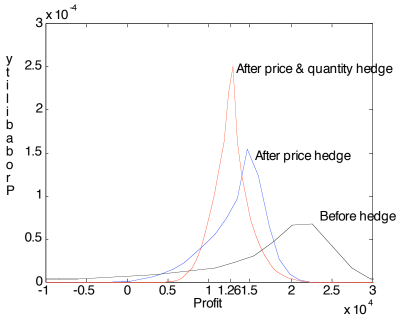


Note: For the mean-variance utility, the optimal payoff is linear in p when correlation is 0,

Volumetric-hedging effect on profit

- Comparison of profit distribution for mean-variance utility (**ρ**=0.8)
  - Price hedge: optimal forward hedge
  - Price and quantity hedge: optimal exotic hedge

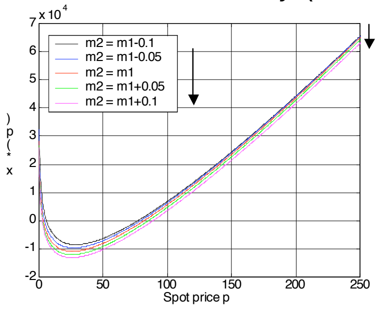
Bivariate lognormal for (p,q)



Sensitivity to market risk premium

$$m_2 = E^Q[\log p] \qquad E^Q[p] = \begin{cases} 63.1, & if\ \ m_2 - m_1 = -0.1 \\ 66.4, & if\ \ m_2 - m_1 = -0.05 \\ 69.8, & if\ \ m_2 - m_1 = 0 \\ 73.3, & if\ \ m_2 - m_1 = 0.05 \\ 77.1, & if\ \ m_2 - m_1 = 0.1 \end{cases}$$

With Mean-variance utility (a =0.0001)

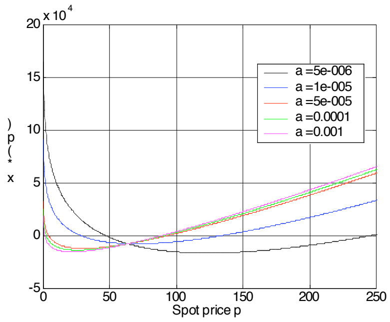


Sensitivity to risk-aversion

(Bigger '*a*' = more risk-averse)

$$E[U(Y)] = E[Y] - \frac{1}{2} a Var(Y)$$

with mean-variance utility (m2 = m1+0.1)



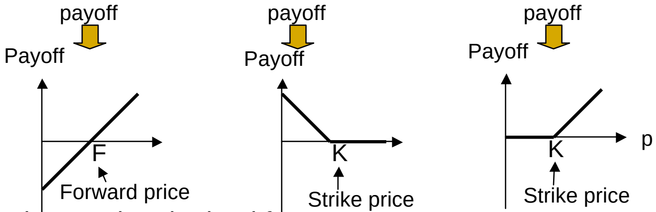
Note: if m1 = m2 (i.e., P=Q), '*a*' doesn't matter for the mean-variance utility.

Replication of Exotic Payoffs

Strike < F   Strike > F

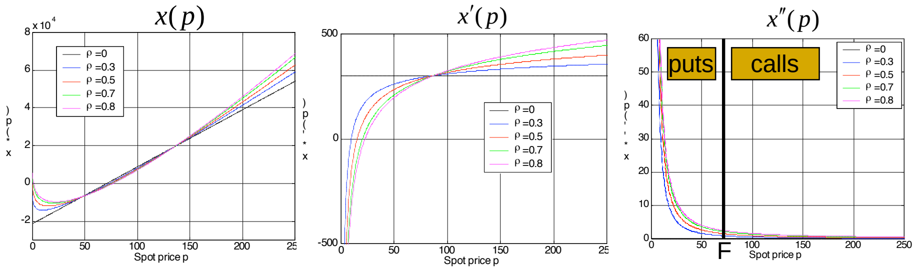
$$x(p) = x(F)\cdot\underline{1} + x'(F)\underline{(p-F)} + \int_0^F x''(K)\underline{(K-p)^+}\,dK + \int_F^\infty x''(K)\underline{(p-K)^+}\,dK$$

Bond payoff | Forward payoff | Put option payoff | Call option payoff

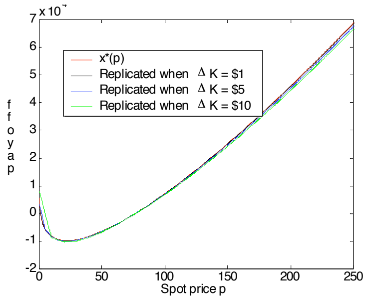


- Exact replication can be obtained from
  - a long cash position of size x(F)
  - a long forward position of size x'(F)
  - long positions of size x''(K) in <u>puts</u> struck at K, for a continuum of K which is less than F (i.e., out-of-money puts)
  - long positions of size x''(K) in <u>calls</u> struck at K, for a continuum of K which is larger than F (i.e., out-of-money calls)

Replicating portfolio and discretization



Payoffs from discretized portfolio



Timing of Optimal Static Hedge

- Objective function

Utility function over profit

$$\max_{\tau} \; E[U((r-p_T)q_T + x_\tau(p_T))]$$

Q: risk-neutral probability measure

$$\text{s.t.} \quad x_\tau(p_T) = \arg\max_{x(p_T)} E_\tau[U((r-p_T)q_T + x(p_T))] \;\; \text{s.t.} \;\; E_\tau^Q[x(p_T)] = 0$$

zero-cost constraint

(A contract is priced as an expected discounted payoff under risk-neutral measure)

$$dp_t = p_t\big(\mu_p(t)dt + \sigma_p(t)dB_t^1\big)$$

(Same as Nasakkala and Keppo)

$$dq_t = q_t\big(\mu_q(t)dt + \sigma_{pq}(t)dB_t^1 + \sigma_q(t)dB_t^2\big).$$

$$EU((r-p_T)q_T + x_\tau(p_T)) = E[(r-p_T)q_T + x_\tau(p_T)] - \frac{1}{2}aVar((r-p_T)q_T + x_\tau(p_T))$$

Example:

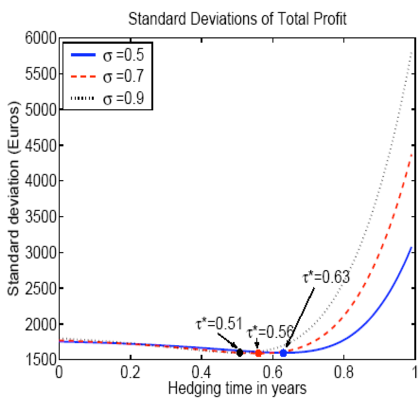
Price and quantity dynamics

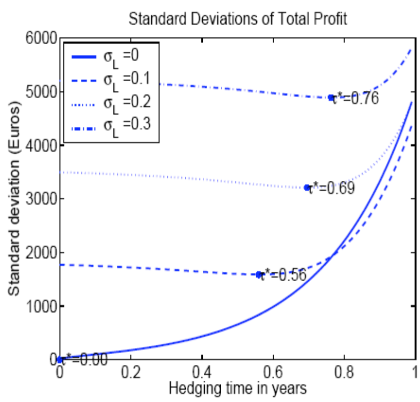
$$\frac{dp_t}{p_t} = e^{-\psi(T-t)}\sigma dB_t^1$$
$$\frac{dq_t}{q_t} = \phi\sigma_L dB_t^1 + \sqrt{1-\phi^2}\sigma_L dB_t^2.$$

The forward price and load estimate for a month one year later is assumed to be 20Euro/MWh and 1000 MWh. The following table summarizes the base values of the parameters:

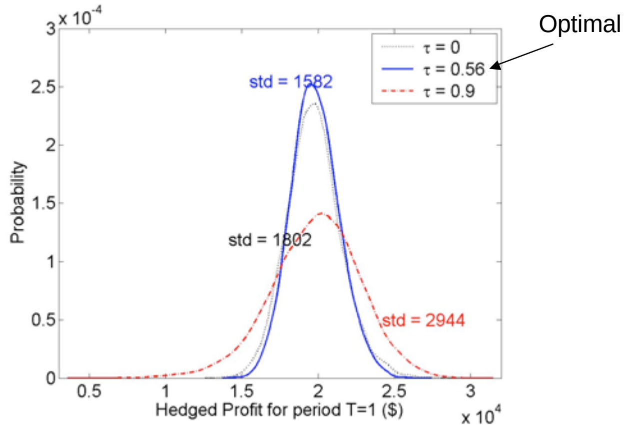
Parameter	T	r	p_0	q_0	ψ	σ	σ_L	ϕ
Value	1	40	20	1000	4.02	0.7	0.1	0.7

Standard deviations vs. hedging time



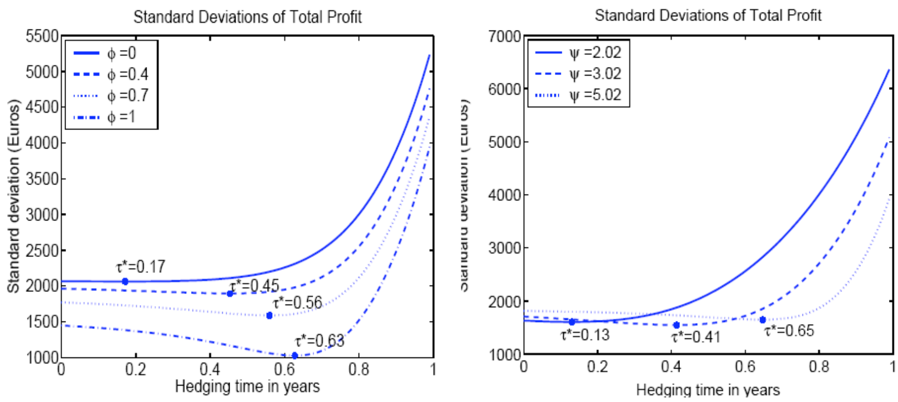


Dependency of hedged profit distribution on timing

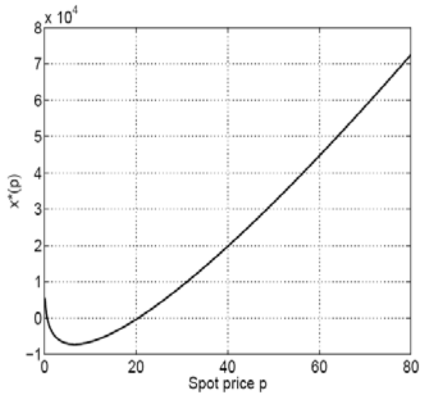


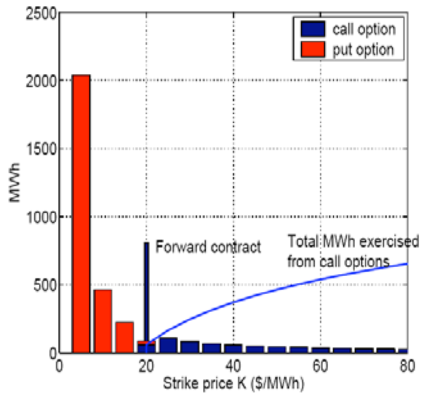
$(r - p_T)q_T + x_\tau(p_T)$

Standard deviation vs. hedging time



Optimal hedge and replicating portfolio at optimal time





*In reality hedging portfolio will be determined at hedging time based on realized quantities and prices at that time.

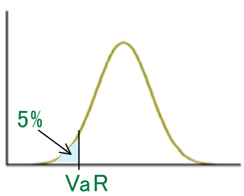
Extension to VaR Contrained Portfolio

$$\max_{x(p)} E[Y(x)]$$

$$\text{s.t. } E^Q[x(p)] = 0$$

$$VaR_\gamma(Y(x)) \le V_0$$

$$\text{where } VaR_\gamma(X) = \nu \text{ such that } P\{X \ge -\nu\} = 1-\gamma$$



- **Let's call the Optimal Solution: x*(p)**

- Q: a pricing measure
- Y(x(p)) = (r-p)q+x(p) includes multiplicative term of two risk factors
- x(p) is unknown nonlinear function of a risk factor
- A closed form of VaR(Y(x)) cannot be obtained

Mean Variance Problem

- For a risk aversion parameter k

$$x^k(p) = \arg\max_{x(p)} E[Y(x)] - \frac{1}{2} kV(Y(x))$$
$$\text{s.t. } E^Q[x(p)] = 0$$

- We will show how the solution to the mean-variance problem can be used to approximate the solution to the VaR-constrained problem

Proposition 1

- Suppose..
  - There exists a continuous function h such that

$$\text{VaR}_{\tilde{a}}(Y(x)) = h(\text{mean}(Y(x)), \text{std}(Y(x)), \tilde{a})$$

with h increasing in standard deviation (std(Y(x))) and non-increasing in mean (mean(Y(x)))

- Then
  - x*(p) is on the efficient frontier of (Mean-VaR) plane and (Mean-Variance) plane

Justification of the Monotonicity Assumption

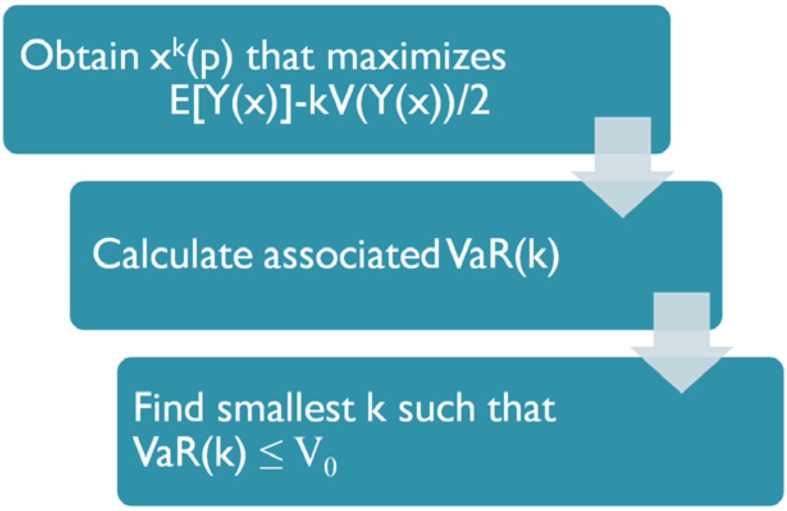
- The property holds for distributions such as normal, student-t, and Weibull
  - For example, for normally distributed X

$$VaR_\gamma(x) = z_\gamma std(x) - E(x)$$

- The property is always met by Chebyshev's upper bound on the VaR:

$$P\{|x - E(x)| \geq k\} \leq var(x)/k^2 \quad \forall x$$
$$\Rightarrow P\{x \leq E(x) - t \cdot std(x)\} \leq 1/t^2$$
$$\Rightarrow VaR_{1-1/t^2}(x) \leq t \cdot std(x) - E(x)$$

Approximation algorithm to Find x*(p)

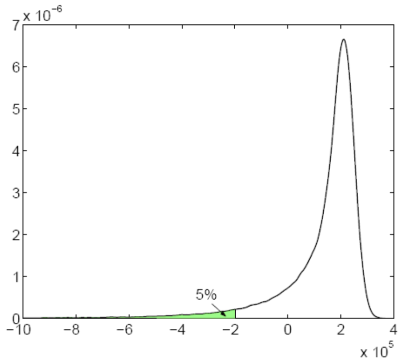


Example

- We assume a bivariate normal dist'n for log p and q

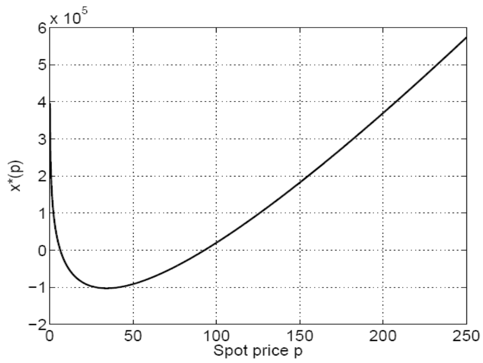
- Under P
  - log p ~ $N(4, 0.7^2)$
  - q~$N(3000,600^2)$
  - corr(log p, q) = 0.8
- Under Q:
  - log p ~ $N(4.1, 0.7^2)$

- Distribution of
  Unhedged Profit (120-p)q



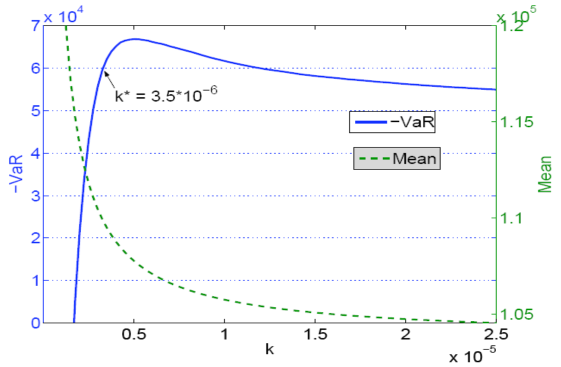
Solution

- $x^k(p) = (1 - Ap^B)/2k - (r-p)(m + E \log p - D) + Cp^B$

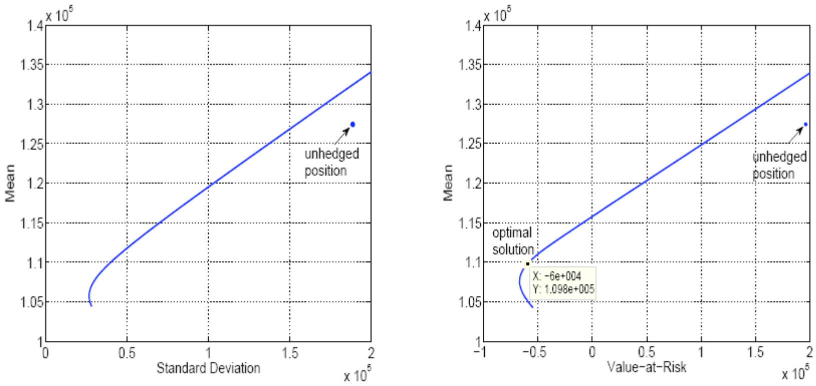


Finding the Optimal k*

- Once we have optimal $x_k(p)$, we can calculate associated VaR by simulating p and q and
- Find smallest k such that VaR(k) $\leq V_0$ (Smallest k => Largest Mean)

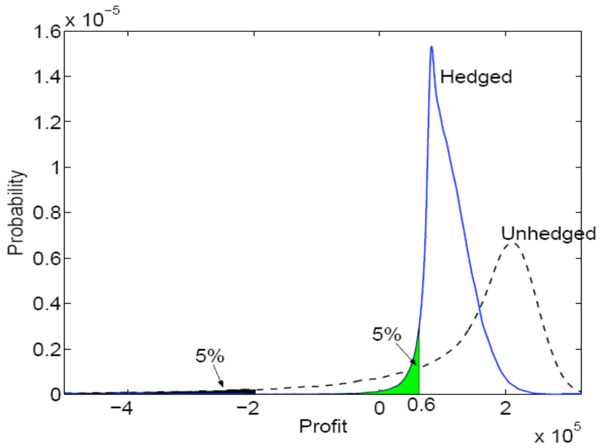


Efficient Frontiers



(a) Mean-variance frontier (b) Mean-VaR frontier

Impact on Profit Distributions and VaRs



Conclusion

- Risk management is an essential element of competitive electricity markets.
- The study and development of financial instruments can facilitate structuring and pricing of contracts.
- Better tools for pricing financial instruments and development of hedging strategy will increase the liquidity and efficiency of risk markets and enable replication of contracts through standardized and easily tradable instruments
- Financial instruments can facilitate market design objectives such as mitigating risk exposure created by functional unbundling, containing market power, promoting demand response and ensuring generation adequacy.