A THEORY OF ELECTRICITY TARIFF DESIGN FOR OPTIMAL OPERATION AND INVESTMENT

R. J. Kaye, Member, IEEE H. R. Outhred, Member, IEEE

University of New South Wales,
Kensington, NSW, Australia

Abstract - This work addresses the problem of balancing supply and demand side operation and investment activities. Existing theory is extended to cover electricity industry models with uncertainty in future conditions and inter-temporal linking such as storage and investment. A new optimal pricing structure which takes these into account is presented. It would induce participants (suppliers and consumers) to make profit maximizing investment and operation decisions which are socially optimal. The structure contains two terms: Short Run Marginal Cost pricing as well as a new "incentive term" to account for the interaction of participants at different time points. A probabilistic forecast of pricing structures at future times is also required. A justification of the result and three simple illustrative examples are given.

Keywords - electricity pricing, demand side options, power system planning, investment under uncertainty, marginal cost pricing

INTRODUCTION

In recent years, electricity supply utilities have been making investment decisions in the face of increasing uncertainty. Many planning parameters have become difficult to estimate due to their volatility. Examples include future values of fuel costs, demand growth, daily load curve shapes, plant construction costs (near and far term) and macro-economic conditions.

In many cases the response has been to re-examine the demand side of the industry. Economic benefits from altering consumers' behaviour and encouragement of private generation and co-generation have been identified. Demand side options (DSO's) can potentially reduce fuel costs by reducing the running time of expensive generators. More significantly, effective exploitation of DSO's can result in large savings on investment costs by delaying or avoiding generation plant installation. It has been widely recognised that consideration of DSO's in the planning strategy can lead to significant cost reductions [1].

The Problem

The question addressed in this paper is the balancing of supply and demand side options. This involves coordinating both operation and investment activity in end use efficiencies and private and supply side generation to achieve an overall optimal result.

A related question arises in supplying the reliability requirements of a variety of different end uses. Here, there is a need to arbitrate between a number of competitors for scarce generation resources.

Recent work (e.g. [2],[3], [4]) has suggested that electricity prices can be used as the coordinating signal. The form of tariff required to encourage optimal demand side behaviour in an uncertain environment has been shown to be a non-predetermined (i.e. spot) price based on the Short Run Marginal Cost (SRMC) of supply.

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The Price-Based Approach

This paper describes a new pricing result which extends the applicability of existing theory to cover both operations and investment in an uncertain environment. The model is designed to accurately represent inter-temporal linking phenomena such as storage and investment which cause inter-dependence of decision making at different times.

An electrical power system is considered to consist of a number of *participants*. Each of these is either a producer or consumer of electrical energy at each time point. The term participant is used to cover the situation where a "consumer" with back-up generation might, if exposed to the appropriate incentive, become a nett producer of electrical energy.

The participants are tied together into an *industry* by the energy balance requirement: at each time, the total electrical energy generated must equal that consumed. For simplicity, transmission and distribution losses are ignored in this paper, although they can easily be incorporated.

A social view of the industry is used to balance the competing requirements of participants. Operation and investment activity which is optimal for the entire industry is characterized. The aim of pricing theory is to cause individual profit maximizing behaviour to be the same as the socially optimal behaviour. In this view, price is the only coordinating signal between participants.

The result presented in this paper indicates that the required pricing structure is SRMC pricing and a new "incentive term" to account for the interaction of participants at different time points. A probabilistic forecast of pricing structures at future times is also required.

This incentive term does not appear in the pricing theory literature of either economics (e.g. [5], [6], [7]) or electrical engineering (e.g. [2], [3]). This is because the current work is based on a more complete model of inter-temporal linkages of decision making under uncertainty. In particular, the price forecasting process is represented in more detail than in other works. It turns out that this has a significant bearing on the final result.

Structure of the Paper

After further defining uncertainty and inter-temporal linking, the social view of an electricity industry is elaborated. This is used to state and justify the optimal pricing result. A rigorous proof is not, however, given in this paper and readers are referred to [8] for details. Three simple examples are then developed. These illustrate the way in which the pricing signal can coordinate both the operation and investment decisions of participants to achieve a socially optimal result. All three are based on simple models but can be extended (at the cost of increased notational complexity) to cover more realistic situations.

INTER-TEMPORAL LINKAGES AND UNCERTAINTY IN AN ELECTRICITY INDUSTRY

Inter-temporal Effects

In many situations, operation and investment decisions at different times cannot be made independently. Past decisions can influence the state of the plant, affecting the current range of decisions available or the plant operating characteristics. Thus decision making must take into account its effect on future plant operation.

Factors which can cause inter-temporal linking of operating decisions by electricity consumers include material storage, thermal storage, product or factor stock piling and plant start-up or shut down sequences. On the supply side, inter-temporal linking is caused by thermal generator start-up times, hydro-electric storages and pumped storage units, amongst other things.

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Investment is a special case of inter-temporal linking, where a decision taken at one time point will affect plant capacity or technical efficiency at some future time, possibly after a long lead time.

Uncertainty

At the time of making an operational or investment decision, there are many phenomena which, although they may affect the future operation of a participant's plant, cannot be known exactly. For example, items of plant break down and are repaired, demand for the outputs of some manufacturing processes change with time and the future of both government policy and macro-economic conditions may be uncertain. There may also be uncertainty in the total cost of an investment and the lead time between an investment decision and its implementation.

A convenient mathematical representation of an uncertain future phenomenon is its probability distribution. However this should be used with caution for processes where the future cannot be thought of as a repetition of the past.

Consequences for Price Setting

The operations of all participants are tied together by the need to maintain energy balance at each time instance, both present and future. Thus, in order to achieve rational coordination of the industry, the pricing signal must contain the appropriate information about future uncertainty and inter-temporal effects of all the participants as well as the immediate state of the industry.

The pricing signal will thus need to induce operation and investment decisions which unknowingly take into account the following:

- A decision can incur costs for other participants and influence their futures.
- Other participants face uncertainty.

Participants on both the supply and demand side should be encouraged to maintain some flexibility to respond to sudden changes elsewhere in the industry such as unexpected growth in system load. For investment decisions would often involve a trade-off between flexibility and economies of scale.

THE SOCIAL VIEW OF PRICE SETTING

The derivation of a pricing policy requires a criterion for balancing supply and demand side operation and investment options. In particular, trade-offs will be required

- amongst the competing benefits of the participants' end uses,
- between the immediate and possible future benefits of electricity usage.

In this paper, the social view of the industry is used to resolve these competing objectives. Each participant is regarded as a profit maximizer and the pricing structure is set to induce a set of operation and investment decisions which are optimal for the entire industry viewed as one economic entity.

Participant as a Profit Maximizer

At the beginning of each time period, each participant is shown the price (more generally, a pricing structure) for that period and a forecast for future periods. It then chooses its operation and investment decisions to maximize its net profit, which consists of two parts: immediately earned profit (i.e. earned by activities in this period) and expected future profits (i.e. earned from activities in future periods).

The immediate profit term can be written as the immediate gross benefit of participation minus the electricity bill for the period. Gross benefit is defined, for an industrial or commercial consumer, as the income from the sale of any product (other than electrical energy) less costs excluding the bill for electrical energy. It allows for the cost of any investment. For a domestic consumer, gross benefit would be the monetary value of the satisfaction derived from the use of electricity less any costs involved.

For a participant who net produces electrical energy, the bill would be negative (i.e. an income) and, in the case of a thermal generator, the gross benefit would allow for the fuel cost.

The second term in the participant's profit, the expected future profits, is the expectation of gross benefits minus electricity bills in future periods. Its consideration is necessitated by the participant's inter-temporal effects (if there are any) which cause future profits to be a function of present decisions. The expectation is integrated over the uncertainty in the participant's own plant as well as uncertainty in the price forecast. Thus the latter will need to represent the uncertainty in all other participants' future operations.

The Social View

The social view of an industry achieves a balance between participants by inducing a set of operation and investment decisions which maximizes the global welfare.

Global welfare is defined as the immediate social benefit plus the expected future social benefits, where the social benefit is the sum individual participants' gross benefits. This excludes payments for or income from the sale of electricity since these are transfer payments within the industry.

The inclusion of the expected future benefits is to account for the inter-temporal linking effects of all participants' decisions onto the future of the entire industry. The expectation is integrated over the uncertainty in all participants' plants.

Optimal pricing theory aims to find electricity prices which cause individual profit maximizing decisions to match global welfare maximizing decisions. In next section, the optimal pricing structure will be defined.

In this paper, it is assumed that participants interact only through the electricity grid: effects such as competing for other scarce resources such as capital are not represented.

By treating the gross benefit of each participant equally, the social view yields economically efficient solutions: investment and operations are directed to where they will cause the most benefit. Thus the optimal pricing signal will resolve demand and supply side trade-offs to the maximum social benefit.

Consider, for example, a situation of supply constraint due to coincident generator failures. There will be a need to curtail some end use satisfaction during the period of constraint. The optimal pricing signal, by inducing socially optimal behaviour, will encourage load reduction from those end uses with lowest gross benefit and from those end uses whose satisfaction can be most cost effectively delayed until the generators have been repaired. Higher variable cost co-generators will also be encouraged to operate.

Similarly, the optimal price should result in a socially optimal solution to the issue of whether to build a new power station or, for example, for the consumers to invest in improved insulation.

WELFARE MAXIMIZING PRICES

The pricing structure which will cause individual profit making decisions to be socially optimal is referred to as the optimal tariff. It is a combination of the well known SRMC pricing and an additional incentive term arising from the effects of inter-temporal linkages. Some re-interpretation of the traditional definition of SRMC [7] is required to account for inter-temporal linking effects and uncertainty.

Definition of SRMC Pricing

Consider an industry with each participant making socially optimal operations and investment decisions so that global welfare is maximized. Suppose that, at one time instance, the entire industry is required to supply one additional incremental unit of electrical energy to a hypothetical consumer who is external to the industry. To achieve this, generation is to be increased and/or load reduced in a fashion which results in the smallest possible reduction in global welfare. SRMC is then defined as the ratio of the reduction in global welfare to the size of the hypothetical external load.

SRMC pricing sets the trading price for each unit of energy to the SRMC. It is thus the purchase price and the buy-back rate.

Although supply of the hypothetical external load is only required for the time instance under consideration, some or all of the actual loss in global welfare might be incurred at some later time due to inter-temporal linking effects. This would appear as a change in the expected future social benefits component of global welfare. For example, if the energy were to be supplied by increased output from a hydro-electric generator, part of the loss of benefit might result from increased thermal generation at a later time. The exact loss of future social benefits would be uncertain because, in this example, the available generation and load demand at future time instances would be not known exactly.

The state of the industry is the condition of each item of plant of each participant. It contains, for example, information about:

- the levels of hydro-electric and demand side storages,
- the state of repair of various components
- amounts of installed and partially installed capital equipment.

The evolution of industry state through time is thus affected by participants' operation and investment decisions and by the outcome of each participant's uncertainty process. Because of the latter, future industry states cannot be exactly predicted; only probability distributions based on the existing state can be estimated.

SRMC is a strong function of industry state and consequently future values cannot be exactly predicted. Thus SRMC prices are not pre-determined like time of use (time of day) prices, but undergo spot variations. Forecasts of future values of SRMC will need to contain probabilistic information.

Mathematically, SRMC is the Lagrange multiplier [9, pp 224-5] associated with the energy balance equality constraint in the global welfare maximization problem [8].

In simpler models, it has been referred to as the "system–λ" [10, pp 23-36]. However, in these models, inter-temporal effects and uncertainty are usually ignored, particularly on the demand side.

In the case of an industry with no inter-temporal effects and no uncertainty, SRMC is either:

* the marginal cost of increasing output from the cheapest available generator that is not producing at full capacity,

or

* the incremental loss of benefit to the least profitable consumer,

whichever is less.

Incentive Term

In some situations, SRMC pricing alone is insufficient. It does not expose participants to the full impact that their immediate decisions might have on future industry state.

One participant's immediate operation or investment decision may, by altering its own future state, affect future values of SRMC. Consider again the example of a hydo-electric generator whose decision to generate now may result in reduced water storage at future times and hence potentially higher SRMC's at those times. Changes in the future values of SRMC will affect the future decisions and hence profits of all participants. It is this effect, which cannot be captured by SRMC pricing, for which the additional incentive term is required.

For a participant j, the incentive term at time t is a function of $d_j(t)$, the operation and investment decisions that j makes at time t. It is defined as $I_j[d_j(t)]$ =

sum of all participants expected future profits if

1. all participants make socially optimal decisions at time t
2. the SRMC and its forecast at the next and subsequent time intervals are based on the industry state that would have resulted if
   - j makes decision $d_j(t)$
   - all other participants make socially optimal decisions

minus

sum of all participants expected future profits if

1. all participants make socially optimal decisions at time t
2. the SRMC and its forecast at the next and subsequent time intervals are based on the industry state that results if all participants make socially optimal decisions

Thus the incentive term exposes a participant to the effects that its decisions might have on future profits of all participants via its effects on future values of SRMC. Note that it is different for each participant and if the participant makes a socially optimal decision it will equal zero. Example 3 below illustrates the calculation of the incentive term.

Main Result

If each participant is shown

* a pricing structure consisting of SRMC pricing and the incentive term defined above
* forecasts for future pricing structures which contain complete probabilistic information (see [8] and Example 2 below)

then the participant will make socially optimal decisions.

Justification of Main Result

The following is not intended as a formal proof. Rather, it is hoped that it will explain the origins of the two terms in the optimal pricing structure.

Consider a participant, j, who is exposed to the optimal pricing structure defined above. At a time, t, it will determine operation and investment decisions, $d_j(t)$, which maximize immediate plus expected future profits.

Suppose j considers a set of decisions $d_j(t)$ and wishes to assess if it is optimal. It will seek small feasible perturbations, $\delta_j(t)$ to that set which result in increased profits. If no such variations can be found, then the profits are maximized.

The perturbation $\delta_j(t)$ will result in changes to j's profits with four components:

$\Delta R_j^1(t)$ = change to j's immediate gross benefits

$\Delta R_j^2(t)$ = change to the SRMC pricing component of the electricity bill.

$\Delta R_j^3(t)$ = change to j's expected future profits due to j's inter-temporal effects on its own future operating conditions. Here, j will assume that its decisions have no effect on future pricing structures.

$\Delta R_j^4(t)$ = change to the incentive term $I_j[d_j(t)]$.

The total change in j's profits resulting from $\delta_j(t)$ is then

$$\Delta R_j(t) = \Delta R_j^1(t) + \Delta R_j^2(t) + \Delta R_j^3(t) + \Delta R_j^4(t) \qquad (1)$$

Now consider the effect that the same perturbations will have on global welfare. There are three components:

$\Delta W^1(t)$ = change to j's immediate gross benefits

$\Delta W^2(t)$ = change to immediate and expected future social benefit by the need to provide (or consume) the extra energy at time t associated with the perturbation $\delta_j(t)$

$\Delta W^3(t)$ = change to future expected global welfare due to change in future industry states resulting from $\delta_j(t)$ via j's inter-temporal effects.

The total change in global welfare resulting from $\delta_j(t)$ is then

$$\Delta W(t) = \Delta W^1(t) + \Delta W^2(t) + \Delta W^3(t) \qquad (2)$$

Clearly, $\Delta W^1(t) = \Delta R_j^1(t)$. For $\delta_j(t)$ sufficiently small, by the definition of SRMC, $\Delta W^2(t) = \Delta R_j^2(t)$, and from the definition of the incentive term, $\Delta W^3(t) = \Delta R_j^3(t) + \Delta R_j^4(t)$. Thus $\Delta W(t) = \Delta R_j(t)$.

Now suppose that the original decisions, $d_j(t)$, were profit maximizing for j. Then for every feasible $\delta_j(t)$, $\Delta R_j(t) < 0$, so that $\Delta W(t) < 0$. Thus no feasible perturbation can be found which increase global welfare so that global welfare is maximized by j's profit maximizing decisions.

EXAMPLE 1: NO INTER-TEMPORAL EFFECTS

The simplest situation is one in which decisions taken at each time period are independent because no participant has any inter-temporal effects. The results of the last section suggest that SRMC pricing should be sufficient to induce socially optimal decisions since the incentive term will be zero.

Industry Model

Consider the operation for one time period of an industry consisting of J participants. For each $j = 1,\ldots,J$, define y_j as the net consumption of electrical energy in that period. Thus

$$y_j = \begin{cases} \text{consumption if } j \text{ is a net consumer} \\ -\text{ generation if } j \text{ is a net producer} \end{cases}$$

During the period, the physical condition of the plant places the following constraints on y_j

$$y_j^{min} \leq y_j \leq y_j^{max} \tag{3}$$

For a nett consumer of electrical energy, $y_j^{min} = 0$ and y_j^{max} is the plant's capacity. For a nett producer, $y_j^{max} = 0$ and $-y_j^{min}$ is the generator's capacity.

Also define $B_j(y_j)$ as the (immediate) gross benefit of participation, so that for a consumer it is the profits before subtracting the electricity bill while for a generator $-B_j(y_j)$ is the fuel costs.

Note that this model could also represent a participant who has the option of being either a nett producer or consumer, an example being a plant with co-generation facilities. In this case, $y_j^{min} < 0$ and $y_j^{max} > 0$.

The energy balance requirement is expressed as $\sum_{j=1}^{J} y_j = 0$.

Socially Optimal Behaviour

The socially optimal decisions, y_j, $j = 1,\ldots,J$, solve

$$W = \max\{ \textstyle\sum_{j=1}^{J} B_j(y_j) : \sum_{j=1}^{J} y_j = 0, \; y_j^{min} \leq y_j \leq y_j^{max}\} \tag{4}$$

The Kuhn-Tucker conditions [9, pp 232-4] for this problem are

For $j = 1,\ldots,J$

$$\frac{\partial B_j}{\partial y_j} - \sigma + \lambda_j = 0 \tag{5}$$

and

$$\lambda_j = \begin{cases} < 0 \text{ if } y_j = y_j^{max} \\ = 0 \text{ if } y_j^{min} < y_j < y_j^{max} \\ > 0 \text{ if } y_j = y_j^{min}. \end{cases} \tag{6}$$

Also

$$\sum_{j=1}^{J} y_j = 0 \tag{7}$$

Equation (6) is the "complementary slackness condition on y_j and λ_j with respect to equation (3)". If the solutions to equations (5), (6) and (7) are $\{\hat{y}_j, \hat{\lambda}_j, j=1,\ldots,J; \hat{\sigma}\}$ then $\{\hat{y}_j, j=1,\ldots,J\}$ solve equation (4) and $\hat{\sigma}$ is the SRMC.

Participant Profit Maximizing Behaviour

Consider any particular participant j, shown a purchase and buy back price of π per unit of electrical energy. In order to maximize its immediate profits, it will choose a nett consumption level y_j to solve

$$R_j = \max\{ B_j(y_j) - \pi y_j : y_j^{min} \leq y_j \leq y_j^{max}\} \tag{8}$$

where $-\pi y_j$ is the bill for electrical energy. The first order necessary conditions are

$$\frac{\partial B_j}{\partial y_j} - \pi + \lambda_j = 0 \tag{9}$$

together with a complementary slackness condition which is identical to equation (6).

If the price, π, is set equal to the SRMC, $\hat{\sigma}$, then since (9) becomes identical to (5), it is clear that $\hat{y}_j$ and $\hat{\lambda}_j$ solve (9). Thus, the individual profit maximizing response is socially optimal.

EXAMPLE 2: SMALL PARTICIPANT INVESTMENT

The following example demonstrates that the forecast of future pricing structures needs to reflect the uncertainty of the entire industry. It also shows that for a small participant whose inter-temporal linking effect on future states of the industry is negligible, SRMC pricing alone induces socially optimal investment behaviour. (In the Example 3, a participant with significant inter-temporal effects will be examined.)

To simplify the exposition, the model consists of only two time periods, labeled t=1 and t=2. During each of these periods, all activity is held at a constant level. After the end of period 2, the industry is assumed to cease to exist and no further activity occurs. However, there is no conceptual difficulty in extending this analysis to an arbitrary number of time periods.

Industry Model

One Consumer: We consider a particular consumer who has a continuous range of investment options in a particular plant item in period 1. This investment has no effect on the operation of the consumer's plant until it is installed at the beginning of period 2. However, the commitment to construct it must be made and paid for during period 1.

For simplicity, assume that, during period 1, this consumer has no activity other than investment.

Define, for this consumer,

c = nett consumption during period 2,

i = cost of investment decision made during period 1.

It is assumed that the cost i completely describes the size and nature of the investment and that it can be chosen from a continuous range. Further define

$B(c,i)$ = gross benefit earned by the consumer in period 2.

Further, assume the consumer is sufficiently small that its operations and investment decisions have no observable effect over the value of the SRMC in period 2. The incentive term will thus be negligible. This will be the situation for most participants in a real power system.

The Remainder of the Industry: Rather than representing each of the other participants, their activity is lumped together into a single remainder of industry model, referred to as "the industry". Again, only activity in period 2 is of interest.

It is assumed that throughout period 2, the industry will be in one of two states, which will be labeled s=1 and s=2. In the former, there is an abundance of cheap generation so that the SRMC, σ_1, is small. When the industry is in state s=2, failure of a number of large generating units has caused the SRMC, σ_2, to be significantly larger than σ_1. Because the consumer is assumed to be small, σ_1 and σ_2 are independent of i and c.

Due to the random nature of the failure and repair of generators, it is not known during period 1 which of these states the industry will be in during period 2. This only becomes known at the beginning of period 2. However, based on the information available during period 1, it is possible to assign probabilities p_1 and $p_2 = 1 - p_1$ that the industry will be in states s=1 and s=2, respectively, during period 2.

Socially Optimal Behaviour

Period 2: Given that an investment decision i was made in period 1 and that the industry in period 2 is in state s (=1 or 2), the socially optimal behaviour for the consumer is to choose c which solves

$$W_2(i,s) = \max\{ B(c,i) - \sigma_s c \} \quad (10)$$

Here, the term $-\sigma_s c$ represents the loss in benefits to other participants arising from the need to supply c units of energy with the industry in state s. The assumption that the consumer is small has been used here.

Period 1: The socially optimal investment i maximizes global welfare which is the sum of the immediate social benefit, which equals $-i$, plus the expected social benefit in period 2. Here the uncertainty is the state of the industry in period 2. Thus i solves

$$\begin{aligned} W_1 &= \max\{ -i + \mathbf{E}[W_2(i)] \} \\ &= \max\{ -i + W_2(i,1)\, p_1 + W_2(i,2)\, p_2 \} \quad (11) \end{aligned}$$

Profit Maximizing Consumer Behaviour

Period 2: Depending on the state of the industry in period 2, a price π will be announced at the beginning of the period. Given that an investment decision i was made in period 1, the consumer chooses c which maximizes its profits i.e. which solves

$$R_2(i,\pi) = \max\{ B(c,i) - \pi c \} \quad (12)$$

By comparing equations (10) and (12) with the price π equal to the SRMC σ_s, it is clear that the profit making decision will be socially optimal in period 2 and that

$$R_2(i,\sigma_s) = W_2(i,s) \quad (13)$$

Period 1: At the beginning of period 1, depending on the projection of the state of the industry in period 2, a price forecast is made. It will take the form of a probability density function (pdf) i.e. $\phi = \{(\pi_1,q_1),(\pi_2,q_2)\}$ where $q_1 + q_2 = 1$. The pdf ϕ is interpreted as meaning that the price in period 2 is

$$\pi = \begin{cases} \pi_1 \text{ with probability } q_1 \\ \pi_2 \text{ with probability } q_2 \end{cases} \quad (14)$$

The profit maximizing response is to choose an investment i which maximizes the sum of the immediate profit, which equals $-i$, plus the expected future profit. Here the uncertainty is in the future price. Thus i solves

$$\begin{aligned} R_1(\phi) &= \max\{ -i + \mathbf{E}[R_2(i)] \} \\ &= \max\{ -i + R_2(i,\pi_1)\, q_1 + R_2(i,\pi_2)\, q_2 \} \quad (15) \end{aligned}$$

If ϕ is set equal to the pdf of the SRMC for period 2, i.e. $\pi_s = \sigma_s$ and $q_s = p_s$ for s=1,2, then, by equation (13), i solves (11). Thus the individual profit maximizing behaviour is socially optimal.

It can be easily shown that the same result cannot in general be achieved using a predetermined price i.e. a single price for period 2 which is announced at the beginning of period 1.

EXAMPLE 3: HYDRO-ELECTRIC POWER SYSTEM

This example demonstrates that, for participants with significant inter-temporal effects, SRMC pricing alone is not sufficient to induce socially optimal behaviour. The incentive term is also required.

The example is based on a simple two period model with no uncertainty phenomena. The industry consists of two participants: a consumer of electrical energy with no storage or investment and a supplier with thermal plant and a single hydro-electric generator. The analysis can be extended to arbitrary numbers of time periods and participants and to include uncertainty on both the supply and demand sides.

Industry Model

The Consumer: The operations of the consumer in each time period (t=1 and t=2) are independent and there are no investment options. Thus its activities can be described by the variable

$c(t)$ = nett consumption during time interval t (=1,2).

The gross benefit of this activity is time invariant and depends only on c(t). Thus define

$S[c(t)]$ = gross benefit (i.e. satisfaction or gross profit) if consumption is c(t)

The Supplier: The thermal generation of the supply side is described by, for t=1,2

$g_T(t)$ = thermal generation during period t

$F[g_T(t)]$ = fuel and variable maintenance costs during period t.

The hydro-electric dam, which is the only source of inter-temporal effects in the industry, is modeled by

$g_H(t)$ = hydro-electric generation during period t

$x_H(t)$ = level of water stored in dam at the beginning of period t measured in equivalent energy units.

Since it is assumed that there is no inflow into the dam, its operation is described by, for t=1,2,

$$x_H(t+1) = x_H(t) - g_H(t) \quad (16)$$

$$0 \le g_H(t) \le x_H(t) \quad (17)$$

The level of water in the dam at the beginning of period 1, $x_H(1)$ is given as data. The operations cost of using the hydro generator is assumed to be zero so that the supplier's gross benefit in period t is $-F[g_T(t)]$.

The Industry: The coordination requirement is the energy balance equation i.e. for t=1,2

$$c(t) = g_T(t) + g_H(t) \quad (18)$$

Socially Optimal Behaviour

During Period 2: Given that the level of the dam at the beginning of period 2 is $x_H(2)$, socially optimal behaviour is c(2), $g_T(2)$ and $g_H(2)$ which solve

$$W_2[x_H(2)] = \max\{ S[c(2)] - F[g_T(2)] : c(2) = g_T(2) + g_H(2)$$
$$0 \le g_H(2) \le x_H(2)\} \quad (19)$$

The Kuhn-Tucker conditions for this problem are

$$S'[c(2)] - \sigma(2) = 0 \quad (20)$$

$$-F'[g_T(2)] + \sigma(2) = 0 \quad (21)$$

$$-\sigma(2) + \lambda_H(2) = 0 \quad (22)$$

as well as equation (18) and a complementary slackness condition on $\lambda_H(2)$ and $g_H(2)$ with respect to (17) (see (6)). Note that S'(.) and F'(.) are the first derivatives of S(.) and F(.) respectively.

Under the assumption that the second derivatives S''(.) and F''(.) are everywhere positive, the solution is

$$\hat{g}_H(2) = x_H(2) \quad (23)$$

$$\hat{\sigma}(2) = -\hat{\lambda}_H(2) = S'[\hat{g}_T(2) + x_H(2)] = F'[\hat{g}_T(2)] \quad (24)$$

$$\hat{c}(2) = \hat{g}_T(2) + \hat{g}_H(2) \quad (25)$$

These will be a function of $x_H(2)$. Note that $\hat{\sigma}(2)$ is the SRMC for period 2.

During Period 1: Given that the level of the dam at the beginning of period 1 is $x_H(1)$ and hydro generation during period 1 is $g_H(1)$ then the level at the beginning of period 2 is according to equation (16), $x_H(2) = x_H(1) - g_H(1)$. Thus, the socially optimal values of c(1), $g_T(1)$ and $g_H(1)$ solve

$$W_1[x_H(1)] = \max\{ S[c(1)] - F[g_T(1)] + W_2[x_H(1)-g_H(1)] :$$
$$c(1) = g_T(1) + g_H(1)$$
$$0 \le g_H(1) \le x_H(1)\} \quad (26)$$

The Kuhn-Tucker conditions for this problem are

$$S'[c(1)] - \sigma(1) = 0 \quad (27)$$
$$- F'[g_T(1)] + \sigma(1) = 0 \quad (28)$$
$$- \sigma(1) + \lambda_H(2) + \sigma(2) = 0 \quad (29)$$

as well as equation (18) and a complementary slackness condition on $\lambda_H(1)$ and $g_H(1)$ with respect to equation (17). Here $\sigma(2)$ solves (24) with $x_H(2) = x_H(1) - g_H(1)$.

Define $\hat{g}_T$ as the solution of

$$S'[\hat{g}_T + \tfrac{1}{2}x_H(1)] = F'[\hat{g}_T] \quad (30)$$

The solution to equation (26) is then $\hat{g}_H(1) = \hat{g}_H(2) = \frac{1}{2}x_H(1)$, $\hat{g}_T(1) = \hat{g}_T(2) = \hat{g}_T$, $\hat{c}(1) = \hat{c}(2) = \hat{g}_T + \frac{1}{2}x_H(1)$ and $\hat{\sigma}(1) = \hat{\sigma}(2) = \hat{\sigma} = F'[\hat{g}_T]$.

Thus the socially optimal solution is to divide activity evenly between the two time periods.

Profit Maximizing Consumer Behaviour

The consumer has no inter-temporal links, so the decisions for each time period can be taken independently. For t=1,2, let $\pi(t)$ be the price for a unit of electrical energy. The profit maximizing consumption c(t) solves

$$R_t^c[\pi(t)] = \max\{ S[c(t)] - \pi(t)\, c(t)\} \quad (31)$$

The first order condition is

$$S'[c(t)] - \pi(t) = 0 \quad (32)$$

Thus if $\pi(t) = \hat{\sigma}(t)$, then by comparing equations (20) and (27) to equation (32), it follows that $\hat{c}(t)$ solves (31).

Profit Maximizing Supplier Behaviour for Period 2

Given that the level of the dam at the beginning of period 2 is $x_H(2)$ and that $\pi(2)$ is the price for a unit of electrical energy, profit maximizing production $g_T(2)$ and $g_H(2)$ solve

$$R_2^S[x_H(2),\pi(2)] = \max\{\pi(2)\cdot[g_T(2)+g_H(2)] - F[g_T(2)] :$$
$$0 \le g_H(2) \le x_H(2)\} \quad (33)$$

The Kuhn-Tucker conditions yield a solution which satisfies

$$F'[g_T(2)] = \pi(2) \quad (34)$$
$$g_H(2) = x_H(2) \quad (35)$$

If $\pi(2) = \hat{\sigma}(2)$, then by comparing these equations to (23) and (24), it follows that $\hat{g}_T(2)$ and $\hat{g}_H(2)$ solves (33).

Profit Maximizing Supplier Behaviour for Period 1

The supplier's hydro generation decision for period 1 affects the amount of water left in the dam at the beginning of period 2 and thus the SRMC, $\hat{\sigma}(2)$, for period 2. Thus SRMC pricing is not sufficient for the supplier in period 1 and the incentive term is required in addition. However, an attempt at using SRMC pricing on its own illustrates the difficulties that arise.

First Attempt Using Only SRMC Pricing: Since there is no uncertainty in the industry, the forecast for $\pi(2)$ at period 1 is its actual value. Thus the pricing information announced at period 1 is $\phi(1) = [\pi(1),\pi(2)] = [\hat{\sigma}(1),\hat{\sigma}(2)] = [\hat{\sigma},\hat{\sigma}]$.

Under SRMC pricing with a dam level of $x_H(1)$, the supplier chooses $g_T(1)$ and $g_H(1)$ to solve

$$R_1^S[x_H(1),\phi(1)] = \max\{\hat{\sigma}\cdot[g_T(1)+g_H(1)] - F[g_T(1)]$$
$$+ R_2^S[x_H(1)-g_H(1),\hat{\sigma}]:$$
$$0 \le g_H(1) \le x_H(1)\} \quad (36)$$

Using equations (33) to (35), the objective function in (36) can be re-written as

$$\hat{\sigma}\, g_T(1) - F[g_T(1)] + K$$

where K does not depend on $g_T(1)$ or $g_H(1)$. While this would induce the socially optimal value for the thermal generation, it is independent of the hydro generation $g_H(1)$.

Thus, under SRMC pricing, the supplier's profit is independent of the amount hydro generation in the first period because the price is the same in both periods and each unit of energy in the storage can be sold for the same amount in either period. The SRMC does not contain sufficient information.

Second Attempt Using Incentive Term and SRMC Pricing: If the supplier uses $g_H(1)$ in period 1, then the water available for period 2 will be $x_H(2) = x_H(1) - g_H(1)$ and the conditional SRMC will be

$$\phi_2[g_H(1)] = F'[g_T^*(2)] \quad (37)$$

where $g_T^*(2)$ solves

$$S'[g_T^*(2)+x_H(1)-g_H(1)] = F'[g_T^*(2)] \quad (38)$$

Here $g_T^*(2)$ is social optimal thermal generation in period 2 if the hydro generation in period 1 is $g_H(1)$.

The incentive term is

$$I_1^s[g_H(1)] = R_2^c\{\phi_2[g_H(1)]\} + R_2^s\{x_H(1)-\hat{g}_H(1),\phi_2[g_H(1)]\}$$

$$- R_2^c\{\phi_2[\hat{g}_H(1)]\} - R_2^s\{x_H(1)-\hat{g}_H(1),\phi_2[\hat{g}_H(1)]\} \quad (39)$$

Using equations (31) to (35), it can be shown that

$$I_1^s[g_H(1)] = S[g_T^*(2)+x_H(1)-g_H(1)] - F[g_T^*(2)]$$

$$- S[\hat{g}_T(2)+\tfrac{1}{2}x_H(1)] + F[\hat{g}_T(2)]$$

$$+ F'[\hat{g}_T(2)]\cdot[g_H(1)-\tfrac{1}{2}x_H(1)] \quad (40)$$

Under incentive and SRMC pricing, the supplier chooses $g_T(1)$ and $g_H(1)$ to solve

$$R_1^S[x_H(1),\phi(1)] = \max\{\hat{\sigma}\cdot[g_T(1)+g_H(1)] + I_1^s[g_H(1)] - F[g_T(1)]$$

$$+ R_2^S[x_H(1)-g_H(1),\hat{\sigma}] :$$

$$0 \le g_H(1) \le x_H(1)\} \quad (41)$$

As before, the objective function in (41) can be re-written as

$$I_1^s[g_H(1)] + \hat{\sigma}\; g_T(1) - F[g_T(1)] + K$$

where K does not depend on $g_T(1)$ or $g_H(1)$. Thus the generation levels to maximize individual profit solve

$$\hat{\sigma} - F'[g_T(1)] = 0 \quad (42)$$

$$\frac{\partial I_1^s[g_H(1)]}{\partial g_H(1)} = 0 \quad (43)$$

As above, equation (42) is solved by $\hat{g}_T(1)$. Differentiating (40) yields

$$0 = S'[g_T^*(2)+x_H(1)-g_H(1)]\cdot[\frac{\partial g_T^*(2)}{\partial g_H(1)} - 1] - F'[g_T^*(2)]\cdot\frac{\partial g_T^*(2)}{\partial g_H(1)}$$

$$+ F'[\hat{g}_T(2)] + F''[\hat{g}_T(2)]\cdot[g_H(1)-\tfrac{1}{2}x_H(1)] \quad (44)$$

The solution to (44) is $g_H(1) = \frac{1}{2}x_H(1)$, which is the socially optimal level $\hat{g}_H(1)$.

CONCLUSIONS

In this paper, an optimal electricity pricing structure has been developed to coordinate the operation and investment activities of suppliers and consumers. The aim is to induce from each participant decisions which are socially optimal so that competing supply and demand side options are compared on the basis of economic efficiency.

The result is that the pricing structure should be set to SRMC pricing plus an incentive term to expose a participant to the effects of its immediate decisions on the future of the industry. Since this results in a non-predetermined tariff, forecasts of future pricing structures are required. The optimal pricing structure excludes any demand terms and Long Run Marginal Cost pricing.

Three simple examples were presented. The first two demonstrated that SRMC pricing alone is sufficient to encourage optimal decisions for an industry with no inter-temporal links or for a participant whose investment and operation decisions did not perceptibly affect future SRMC values. The third example showed that this was not true for participants with significant inter-temporal linkages.

This paper does not address the significant problems associated with implementation. This will require major advances. Some preliminary results are contained in [4].

The results described in this paper can be extended to cover industries with transmission and distribution losses and to the pricing of competing energy forms such as oil, gas and electricity. Discounting of money values across time periods can also be easily incorporated.

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R. John Kaye obtained his B.E. and M. Eng. Sc. degrees from the University of Melbourne in 1977 and 1979, respectively. After working for the State Electricity Commission of Victoria, he graduated with a Ph. D. in electrical engineering and computer sciences from the University of California at Berkeley in 1982. After completing a post-doctoral fellowship at the Australian National University, he joined the University of New South Wales where he is currently a lecturer in Electrical Engineering and Computer Science. His research interests include power system economics and planning.

Hugh R Outhred holds the degrees of B.Sc., B.E. and Ph.D. all from the University of Sydney. He worked briefly for the Electricity Commission of New South Wales before joining the Department of Electric Power Engineering at the University of New South Wales. He has also worked for the Energy Authority of New South Wales as a consultant and as acting head of the Forecasting Branch. His current research interests are in the areas of energy planning, energy economics, demand management and renewable energy.

Discussion

Robert H. Sarikas (Foster Associates, Inc., Washington, DC): The analysis, while not explicitly stated in the paper, assumes that the cost of capacity additions are divisible, i.e., can be purchased in the desired increments in order to match capacity requirements. It also appears that there is an implicit assumption that the capacity mix can be optimized without regard for the life-span of typical generating facilities.

In the real world, it is frequently impossible to size new units to exactly meet load requirements due to the so called "lumpiness" of capacity additions. Also with respect to optimization, planners can only optimize capacity additions, in the context of an expansion plan. The existing system cannot be modified. Ralph Turvey states this clearly:

> An equally platitudinous consideration which needs to be borne in mind is that public enterprises exist. They can be expanded or contracted, but it would be wasteful to build a new one from scratch this year if an existing one has been inherited from last year. Hence, talk about how costs would behave as a function of the size of a brand new one is pointless. This means that textbook longrun industry cost curves which reflect only today's technology and today's factor prices are no use. What matters are the costs of running and of expanding or contracting the hodge-podge we have got, the fossilization of past decisions taken by our predecessors.[1]

In view of the aforementioned assumptions, the solution described in this paper, a very worthwhile first step, does not appear to meet real world conditions. Is it possible to extend the analysis so as to mitigate the impact of those implicit assumptions?

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R. J. Kaye and H. R. Outhred: We thank Dr. Sarikas for his considered discussion of our paper. His comments are most interesting.

One of the main weaknesses of the pricing theory presented in the paper is its inability to deal with indivisible investments. All decisions are assumed to be drawn from a continuous range, which as Dr. Sarikas observes, is not the case for supply side investment decisions. Here, the options available to the planner will often contain discrete choices, such as the number of units of each available generating technology.

This weakness is common to almost all advanced pricing theories: it is an open question as to whether there is a pricing signal which encourage socially optimal indivisible investment decisions. Work on this question is currently underway at the University of New South Wales. While it is to early to supply a concrete answer, one proposal is outlined in [1].

However, we do not agree with the suggestion that the current work disregards life-spans of typical generating facilities. The theory is based on a dynamic state representation of each participant's plant and thus all such phenomena can be represented.

Details of the state model are available in the paper's reference [8]. For each participant, the current values of all phenomena involved in intertemporal linking of decisions are part of the state vector. This evolves dynamically over time as a function of the outcome of the uncertainty processes and the participant's operation, investment and retirement decisions. For example, components of the state vector might include:

- thermal or material storage levels
- an indication of the progress through a start up or shut down procedure
- the installed capacity and age of each item of plant
- a measure of the condition of each item of plant, including any deterioration incurred through, for example, previous adverse operations.

Thus each participant's state will include all available information which impinges on the life of the plant.

Socially optimal decisions are state contingent: they are a function of the existing state of each participant's plant. Thus investment behaviour under optimal pricing (which is also a function of the states of all participants) will account for all factors affecting each participant, including anticipated plant life times.

The theory presented in this paper is quite general. The major assumption is the divisibility of investments which is particularly severe on the supply side.

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