Border flow rights and Contracts for differences of differences: Models for Transmission Property Rights

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Abstract

In this paper a property rights model for electric transmission is proposed and its properties analyzed. The proposed rights, called "border flow rights," support financial hedging of transmission risk and merchant transmission expansion through associated financial rights, called "contracts for differences of differences." These financial rights allow for short-term and long-term trading of both energy and transmission in an exchange.

Keywords

Electricity market, Property rights, Transmission investment, Financial transmission rights, Energy and transmission trading.

I. Introduction

This paper builds on recent work by Gribik *et al.* [1], [2] that describes long-term property rights for transmission expansion and on provisions in the Australian electricity code [3]. We propose a property rights model for existing and new transmission investment, called "border flow rights." By a property rights model for transmission, we mean a definition of an underlying revenue stream that accrues to the owner of a transmission line. In particular, under the border flow rights model, the owner of a transmission line or lines is paid the locational marginal price for energy that it delivers to the rest of the system and pays the locational marginal price for energy that it receives from the rest of the system. The border flow rights model therefore values transmission by its contingency-constrained transport of lower value energy to higher value locations.

Border flow rights accommodate trading of financial transmission congestion hedging instruments called "contracts for differences of differences" (CFDDs). Trading based on CFDDs is reminiscent of "contract path" transactions but, unlike the contract path mechanism, incorporates Kirchhoff's laws and represents contingency constraints. CFDDs can be arranged between parties analogously to "contracts for differences" (CFDs) [4, §3-2.1] that are used to hedge locational marginal price variation at a given location.

CFDDs define a financial transmission right (FTR) that, in principle, can be traded without a central exchange. This is unlike the auctioning of FTRs in current markets that require the Independent System Operator (ISO) to be intimately involved in the forward trading of transmission. The ISO need not be involved in trading of CFDDs because revenue neutrality for the ISO is guaranteed by the border flow rights model. This feature avoids the need for ISOs to be involved in FTR trading, contrasting with current formulations of transmission property rights including that in Gribik *et al.* [1], [2].

Although contracts for differences of differences can, in principle, be traded without an exchange, such an exchange is likely to help in matching providers of transmission services with users of transmission services, having a role that is somewhat more extensive than the role of a clearing-house for energy contracts. The exchange could facilitate trade of *both* forward transmission and forward energy contracts simultaneously over various timescales. This would avoid the current separation of forward trading of transmission, by ISOs, from forward trading of energy, which is generally not performed by ISOs.

In the context of energy, CFDs allow the co-existence of short-term offer-based economic dispatch by the ISO with longer term financial contracts to establish forward financial positions, without either constraining

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the other. In contrast, currently implemented FTR mechanisms require the ISO to be intimately involved in the allocation and reconfiguration of forward transmission rights because congestion rental accruing to the ISO funds the FTRs. Contracts for differences of differences, as proposed in this paper, would allow the ISO to instead focus solely on short-term dispatch, while forward financial positions could be established without ISO involvement and without ISO financial exposure.

The organization of the paper is as follows. Section II contains a brief literature survey and contextualizes the goals of the paper. In section III, we discuss the hedging of transmission and energy prices. In section IV, we discuss the remuneration of transmission investment, with section V presenting an example. Section VI discusses the proposed financial hedging mechanism, the relationship to the contract path paradigm, and option rights. Alternatives for trading of the rights are sketched in section VII using the example from section V to illustrated the alternatives. Merchant transmission is discussed in section VIII, with section IX concluding. An Appendix establishes some theoretical results that simplify and generalize the analysis in Gribik *et al.* [2] and apply the results to an example system.

II. Literature survey and goals

Financial transmission rights have been defined and discussed in several papers. For example, see Hogan [5], [6], [7], Chao and Peck [8], [9], Oren [10], Bushnell and Stoft [11], [12], [13]. Point-to-point financial transmission rights, as typically defined as being issued by the ISO, must be reconfigured centrally in order that the collection of rights satisfies the "simultaneous feasibility test" (SFT). The first goal of the present paper is to remove the need for the ISO to be the issuer of rights, allowing for reconfiguration of transmission rights by an entity other than the ISO.[1] This goal is achieved by defining transmission property rights in terms of an underlying revenue stream.

A second goal is to remove the risk to the ISO of revenue shortfall under transmission outage conditions. The risk is devolved to the owners of the transmission assets.

The third goal of the present paper is to define a property right and associated financial right that supports merchant transmission expansion. Hogan discusses merchant based transmission expansion in [15], while Joskow and Tirole present various problems with merchant expansion in [16].

We make no attempt to solve all the problems associated with merchant transmission that are described in [16]. Experience with merchant transmission in Australia is described in [17], [18], while experience with merchant transmission in Argentina is described in [19], [20]. Joskow describes transmission policy in the United States in [21].

Moreover, the main focus in the paper is on energy rather than on reserves or reactive power. We do not directly consider the value of transmission in enhancing reliability by the sharing of reserves, except as it affects the contingency-constrained capacity of the network to transport energy. We do not directly consider reactive power. The formulation can be expanded to include explicit representation of reserves and reactive power and payment to transmission for "transporting" reserves and reactive power.

Finally, we observe that we omit discussion of market power in both the generation and transmission markets, but appreciate that it can be a problematic issue, with important inter-relationships between the energy and transmission prices. In particular, we do not consider the value of transmission in mitigating locational market power. See, for example, [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32]. In particular, see [27] for cases where a line with small capacity and zero flow has an important role in mitigating market power in two inter-connected markets.

III. Hedging of transmission and energy prices

From a normative perspective, pricing of transmission services to customers should incent the efficient use of scarce transmission capacity. It is well known that locational marginal prices (LMPs) provide efficient

[1] Although the ISO need not participate in the reconfiguration of the transmission rights, the ISO is still assumed to perform centralized offer-based security-constrained dispatch. That is, the dispatch function is not decentralized. Approaches to decentralized dispatch in the presence of congestion are discussed in [14].

incentives for generators and consumers and for transmission use [33]. In contrast, “contract path” pricing of transmission services usually provides an inefficient signal to users of transmission services.

Since LMPs and LMP differences are volatile, however, market participants typically desire financial instruments to hedge against the variation in LMPs and LMP differences. In the absence of transmission constraints and ignoring losses, CFDs can be used to hedge LMP volatility. However, when there are binding transmission constraints, CFDs alone cannot hedge a transaction from a generator to consumer that are not co-located, since the LMP at the generator and consumer will differ.

Financial transmission rights (FTRs) [7] are in use in several electricity markets to hedge the volatility of LMP differences when there are transmission constraints. Such rights are either allocated to pre-existing transmission rights holders or sold through a centralized auction or sequence of auctions.

In all existing market implementations, the auction for FTRs is conducted by the ISO, which sells the rights to bidders on the basis of their willingness to pay for the transmission rights. A purchaser of a “point-to-point” FTR receives, over the contract duration of the FTR, the right to a revenue stream from, say, the day-ahead market, equal to the FTR contract quantity multiplied by the LMP differences between the points of injection and points of withdrawal.[2] This revenue stream is paid from the congestion rental accruing to the ISO from offer-based security-constrained dispatch (OBSCD).

To ensure that the congestion rental is *adequate* to cover the payments to FTR holders, the allocated and sold rights must satisfy the SFT. That is, the allocated rights together with the sold rights must satisfy the transmission constraints, including contingency constraints, in order to ensure “revenue adequacy.” (See [5], [36], [37] for formal proof of this result.) The involvement of the ISO in the auction and the use of the SFT is predicated on the assumptions that:

1. the FTRs are paid out of the congestion rental accruing to the ISO in the OBSCD and
2. the ISO should remain on net, at least approximately, revenue neutral, with congestion rental from the OBSCD at least covering its obligation to FTR holders.

In later sections, we will change the first assumption and in so doing create a transmission rights mechanism that is *exactly* revenue neutral for the ISO.

Transmission rights are often sold for durations that are much longer than a day, during which time some of the transmission lines represented in the SFT “test system” may actually be out of service. When lines are out of service, the revenue adequacy of the issued FTRs will not necessarily hold true. The ISO has several alternatives under these circumstances, such as:

1. It can assume some risk of revenue shortfall (presumably charging it as an “uplift” or averaging the shortfall from pricing periods when there is an outage against other periods of positive net revenue),
2. The ISO can implement a derating policy or scale down the FTR payments, or
3. The ISO can deliberately sell less rights than are implied by the SFT.

The first alternative reduces the value of performing the simultaneous feasibility test, which is fundamentally to prevent revenue shortfall. The second alternative blunts the ability of the FTR to hedge transmission charges, while the third means that some transmission capability is not being offered to the market. That is, none of these alternatives are entirely satisfactory in the context of hedging LMP differences.

Furthermore, if a purchaser of point-to-point FTRs finds that its needs change, it will generally not be able to sell the right completely “over the counter” to another party, unless the other party requires an FTR between electrically very similar points.[3] Consequently, relatively frequent auctions are required to enable reconfiguration of the transmission rights as transmission needs change.

To summarize, to ensure revenue adequacy the ISO must only sell transmission rights that collectively satisfy the simultaneous feasibility test, given an assumed test system. During periods when the actual capability falls short of that in the test system, the ISO must have some policy for either covering the revenue shortfalls

[2] Alternative financial rights include “flowgate” rights [8], [34] and “contingent” rights [35]. Hogan describes some difficulties with flowgate formulations in [7].

[3] In some formulations, hubs are defined to allow a point-to-point FTR to be specified as the sum of a “point-to-hub,” a “hub-to-hub,” and a “hub-to-point” right. In this case, the “hub-to-hub” right is more likely to be easily tradable; however, the “point-to-hub” and “hub-to-point” rights remain specific to the purchaser, requiring a central auction for reconfiguration.

or derating the system or must otherwise undersell the capability to minimize the likelihood of failing to be revenue adequate. Moreover, the ISO must repeat the auctions on a regular basis to enable reconfiguration of the rights.

These requirements for FTR auctions contrast greatly with financial hedging of energy bought and sold at a single bus. A CFD can be arranged between generators and consumers (or a load serving entity purchasing on behalf of consumers) or through an exchange that is not associated with the ISO. CFDs hedge the volatility of LMPs at a single location and enable financial bilateral contracts in the context of OBSCD. Generators and consumers have opposite tastes for exposure to LMP fluctuations: a high price is "good" for a generator and "bad" for a consumer and vice versa. Consequently, each can costlessly hedge risk by signing a CFD.

CFDs can be arranged without any intervention by the ISO and without the CFD posing any revenue risk to the ISO in the event of generation outages.[4] They can be arranged for short durations to support short-term opportunities or for long durations to support the financing of new investment. In practice, the convenient matching of generators and consumers is likely to benefit from a public exchange. For example, [37] describes a model of a sequence of auctions that can be used to arrange such financial contracts. However, there is no material need for the ISO to be involved in trading of CFDs. The rest of this paper is aimed at the development of a property right definition for transmission and an associated financial mechanism that does allow for decentralized trading of transmission services and which is analogous to CFDs but applied to transmission.

IV. Remunerating transmission investment

From a normative perspective, the remuneration of transmission investment should incentivise efficient transmission investment decisions. This implies that the property right conferred upon an investor in new transmission should produce a revenue stream that incents the efficient level of investment.

In sections IV-A–IV-C, we discuss remuneration schemes that involve financial transmission rights, flowgate rights, and remuneration based on the sensitivity of welfare.

A. Financial transmission rights

FTRs have been proposed as a property right that, under some assumptions, provides efficient incentives for new construction. Under the FTR mechanism, the builder of a line *nominates* an FTR that, together with the other allocated and sold rights, is feasible for the newly expanded system [15], [38].[5] (Such a rights holder might then offer its capacity into the ISO reconfiguration auction to sell to transmission users, possibly involving reconfiguration of this and other rights into a different point-to-point right.)

Bushnell and Stoft [11], [12], [13] show that if an investor is rewarded with such a right then, under the restrictive assumption that the implied operating point for the SFT is the same as the actual operating point for the OBSCD, the investor is incentivised to make only beneficial modifications to the network because the payment to the investor matches the benefits to the system. Hogan also discussed incentives for investment in [15].

In fact, it is unlikely that the operating point for the SFT will be the same as the actual operating point of the OBSCD because of diurnal and seasonal variation in demand and the resulting variation in patterns of dispatch over the contract duration of the FTR. If patterns of actual dispatch turn out to be different to that implied in the SFT then the value of the FTR might be relatively small, even if the transmission project itself conferred significant benefits to the system. The ISO will accumulate some of the congestion rental under these conditions. Consequently, it may not be possible to nominate an FTR that will provide payment to the builder that reflects the value of the line to the system. The implication is that the builder will not, in practice, be incentivised to make the most beneficial modifications to the network.

[4] Of course, there is risk to the ISO of parties to a CFD failing to pay the ISO for their energy based on nodal prices. However, this risk is not due to the CFD itself.

[5] A complication with this scheme is that there may be some unallocated and unsold capacity still "on the table" before the transmission is built. In this case, we may prefer to only allow the investor in new transmission to obtain rights to capacity that it actually contributed. This significantly complicates the nomination process, as discussed in [38], [39], [40], [41].

For an extreme example of the implications of the requirement of FTR nomination, suppose that a new line was proposed to join two regions, A and B. Suppose that prices were anticipated to be lower in A than in B most of the time. Therefore, the builder would presumably nominate an FTR from A to B.

However, suppose that the LMP was higher at A than at B for a significant fraction of time. This could be because the flow turned out to be from B to A during these times. The nominated FTR from A to B would have *negative* value, but the transmission project itself might produce considerable value by allowing transmission from B to A.[6]

A partial solution to this problem is to perform a sequence of auctions with relatively short contract periods and to allow re-nomination of FTRs for each contract period [38, Page 17]. For example, there could be an auction for each season. However, seasonal nominations do not completely solve the problem unless the season correlates strongly with the direction of flow.

To summarize, the value of the FTR depends critically on the nomination of the FTR by the builder and does not evidently allow the builder to capture the value of transmission flows that were not anticipated in the FTR nomination or capture the value of flows that are very different to that modeled in the SFT.[7] FTRs do not reflect the "underlying" value of the line to transmit power. They reflect only the value under dispatch conditions that are similar to the SFT test system.

Even putting aside the problem of nomination of the FTR, there is another issue that is both the principal advantage and also a problem with FTRs. Nomination of FTRs assigns all the benefits of incremental transmission to the FTR nominator. That is, the previously "unused" capacity of other lines in the system are effectively appropriated by the FTR nominator. This means that the nominator is motivated to propose upgrades that maximize overall benefits to the system. However, the owners of other lines have no incentive to provide or maintain their capacity in the face of such a mechanism because they do not benefit from it.

The right conferred by an FTR should be compared to the property right conferred on a generator that interconnects with the system. In an offer-based pool, such a generator is deemed to have the right to a revenue stream equal, at each time, to its energy generation times the LMP at its bus. The *financial* counterpart of this revenue stream is a financial energy contract at the bus, such as a CFD; however, it is important to note that the *underlying* revenue stream paid to the generator does not depend on any financial nomination such as a CFD: the underlying revenue stream of LMP times energy is conferred upon the generator for its production of energy whether or not it is party to a CFD.

In contrast, consider the revenue stream accruing to an FTR that is nominated by the builder of the line. The revenue stream depends explicitly on the FTR nomination as well as on the intrinsic line characteristics. In the case of FTRs there is no underlying revenue stream for transmission services that is analogous to a generator being paid LMP times energy. Unlike the case of a CFD, one cannot directly observe the value of an FTR in terms of a revenue stream to an asset that is *independent* of a financial nomination.

To summarize, awarding FTRs to incremental transmission investment creates an asymmetry between the right conferred on a transmission investor and the right conferred on a generation investor. Unlike the case for a generator, in the case of transmission there is no underlying revenue stream that is independent of the financial rights nomination. The situation is illustrated in table I, which compares current implementations of energy and transmission markets. As shown in table I by the question marks, there is no underlying revenue stream for transmission assets defined in current implementations of energy and AC transmission markets.[8]

[6]This situation is not merely theoretical. For example, in the case of the interconnection between the electricity systems of the states of New South Wales and Queensland in Australia, it was initially anticipated that power would be exported from New South Wales to Queensland. However, power has often flowed from Queensland to New South Wales [18, page 3 and footnote 7]. Similarly, flows on the line from Almafuerte to El Bracho in Argentina have been in the opposite direction to the initial expectation [20, page 7]. In the presence of merchant generation, there are likely to be unanticipated patterns of power flow.

[7]"Bi-directional" option FTRs can, in principle, solve this problem; however, it is an open question as to how to appropriately represent options in the SFT in a way that is computationally tractable and does not severely limit the amount of FTRs that can be sold [6]. We will see in section VI-C that option instruments can be constructed easily with the financial rights we propose in this paper.

[8]There already exists such an underlying revenue stream for DC transmission, namely the difference between the energy bought and the energy sold. The border flow right proposed in this paper is consistent with this revenue stream.

TABLE I

COMPARISON OF CURRENT IMPLEMENTATIONS OF ENERGY AND AC TRANSMISSION MARKETS.

Asset:	Product:	Underlying revenue stream:	Financial instrument:
Generation	Energy	Energy $\times$ LMP	Contract for differences
Transmission	Transmission	???	Financial transmission right

B. *Flowgate rights*

An alternative rights mechanism involves conferring flowgate rights for the contributed capacity of a new line. This right can be defined in terms of an underlying revenue stream. In particular, the property right for a flowgate could be defined to be the right to receive the product of the capacity of the flowgate multiplied by the shadow price on this capacity constraint in the OBSCD.

The revenue stream for a flowgate as just defined is independent of any financial nomination and so is more satisfactory from this perspective. It is also possible to sell portions of the capacity of a flowgate "over the counter" for different durations without complications.

However, by design, this will assign the benefits of new capacity to the congested line. This will again provide no incentive for other lines in the system to maintain their capacity.

C. *Sensitivity based rights*

To remedy the deficiency of flowgate rights, Gribik *et al.* [2] propose a financial right that depends on both the capacity and the electrical admittance of the line. Gribik *et al.* suggest a financial transmission right that involves payment based on the sensitivity of welfare to both capacity and admittance.[9]

The border flow right we define has a similar character to that proposed in Gribik *et al.* [1], [2] in that the underlying revenue stream for the border flow right is based on a similar sensitivity calculation. However, in section VI we will use this underlying revenue stream to define a financial right. Because of its relationship to the sensitivity of welfare, the border flow right provides incentives to build transmission that are analogous to the guidance provided by sensitivity based transmission planning as described by Dechamps and Jamoulle [42] and Pereira and Pinto [43].[10] In particular, since the underlying revenue stream for the border flow right is based on the sensitivity of welfare, we will see that the border flow right incents efficient *marginal* transmission expansion by coalitions of beneficiaries.[11]

Moreover, by conferring payment on parallel lines and not just on the lines with binding capacity constraints, the payment mechanism more directly encourages competitive suppliers of transmission. For example, consider a line with flow that is less than its capacity, but for which there is a positive difference in LMPs between its ends. Under the right proposed by Gribik *et al.*, such a line receives payment and can also

[9] Gribik *et al.* demonstrate the case for both real power and reactive power prices, but we will restrict our discussion to real power. The principle in Gribik *et al.* can be generalized to any vector of quantities that are conserved nodally and to any parameters that enter into the power flow equations and the constraints *linearly,* when both the base-case and contingency-case constraints are represented explicitly. For example, one could imagine a payment scheme based on separate prices for real power, reactive power, real power reserves, and reactive power reserves using, for a transmission line, the sensitivities to susceptance, conductance, and line flow limits. Payment to a phase-shifting transformer would be based on the sensitivity to susceptance, conductance, and tap-changer limits. Each of the quantities of real power, reactive power, real power reserves, and reactive power reserves is conserved nodally and each of the parameters of susceptance, conductance, line flow limits, and tap-changer limits enter into the power flow equations and constraints linearly. The development in the appendix accommodates these generalizations. If the parameters appear *non-linearly* in the power flow equations or constraints then a similar sensitivity based scheme will have the same incentive properties; however, such a scheme will no longer be revenue neutral for the ISO.

[10] I am indebted to Professor Rubén Darío Cruz of Universidad Industrial de Santander, Colombia, for bringing this interpretation to my attention.

[11] As with all marginal schemes, the presence of scale economies and lumpiness may lead to insufficient capacity being built. In addition to the problems due to scale economies and lumpiness, transmission expansion is also complicated by the "network externalities" due to Kirchhoff's laws. The right proposed by Gribik *et al.* and the border flow right defined here deal correctly with the network externalities, assuming that the OBSCD represents the network faithfully; however, it does not solve the issues raised by scale economies and lumpiness. For an empirical study of scale economies in transmission construction, see [44].

increase its revenue stream by increasing its admittance and so attracting more power flow on it. By doing so, such a line will be allowing more power to be transferred from nodes with low prices to nodes with high prices and will therefore be increasing welfare.[12]

C.1 No contingency constraints

If we first ignore contingency constraints then the underlying revenue stream we propose for an owner of a transmission line joining nodes k and ℓ is given by:

$$p_k P_{\ell k} + p_\ell P_{k\ell}, \tag{1}$$

where:

- the LMPs at busses k and ℓ are p_k and p_ℓ, respectively,
- the power flow from the line into bus k is $P_{\ell k}$, and
- the power flow from the line into bus ℓ is $P_{k\ell}$.

This payment is equal to the congestion rental on a line in a system consisting of a single radial line between busses k and ℓ. However, we are proposing this payment for all lines, even in non-radial systems. As in Gribik *et al.* [2], we can then interpret the revenue stream as a redistribution of the congestion rental.

The equivalence between payment based on sensitivity of welfare and payment based on (1) is proved in the appendix, simplifying and generalizing the development in Gribik *et al.* [2]. The LMPs and the flows on the lines are determined by the ISO as the result of an OBSCD. The payment is made in each pricing period based on the LMPs and the flows for that pricing period. (For clarity, we have suppressed the explicit dependence of LMPs and flows on time.) Since $P_{k\ell}$ and $P_{\ell k}$ are generally of opposite sign and since positive flow from the line into the bus will usually be at the higher price bus, the revenue stream is usually positive. Cases where the revenue stream is negative are discussed in section VI-C.[13]

The revenue stream defined in (1) is analogous to the revenue stream paid to a generator for generation and paid by a consumer for its consumption. In particular, a generator is paid at the LMP for its generation, a consumer pays at the LMP for its consumption and, according to (1), a transmission line is paid at the LMP for energy that it delivers to the system and pays at the LMP for energy it receives from the system. Moreover, the ISO is *exactly* revenue neutral under this payment scheme under all dispatch conditions because power is conserved nodally. That is, all the power entering a bus also leaves that bus. Consequently, since all energy is bought and sold at the LMP, the net revenue to the ISO is exactly zero by definition. (See the Appendix for proof.)

The proposed revenue stream to transmission owners is in contrast with the situation where only generation is paid at the nodal price and only consumers pay at the nodal price. In the latter case, the congestion rental accrues to the ISO. As discussed in section III, the ISO must then pay out the congestion rental in order to make it approximately revenue neutral leading to complications if the issued financial rights do not happen to correspond to a secure dispatch for the system.

C.2 Contingency constraints

In the case where contingency constraints are binding, an exact calculation of the sensitivity of welfare implies a payment involving the sum over normal and contingency states of the product of net injection or withdrawal in a state times an appropriate state dependent LMP [1, Appendix B]. This result is also proved in the appendix. In simple cases, the payment can be evaluated with (1) using the flows on the lines calculated for the contingency cases. This payment scheme is also revenue neutral for the ISO.

[12] It is interesting to note that admittance is "always on the margin" in the sense that there will be a non-zero price for admittance if there is at least one transmission constraint binding in the system, whereas capacities of particular lines are not always limiting. I am indebted to Richard O'Neill of the Federal Energy Regulatory Commission for bringing this interpretation to my attention.

[13] Interestingly, although (1) and variants of it are well-accepted as appropriate ways to charge transmission customers for *use* of transmission services and to remunerate owners of DC transmission [3][18, footnote 64], Gribik *et al.* seem to be the first authors to propose this scheme for remunerating AC transmission *owners*.

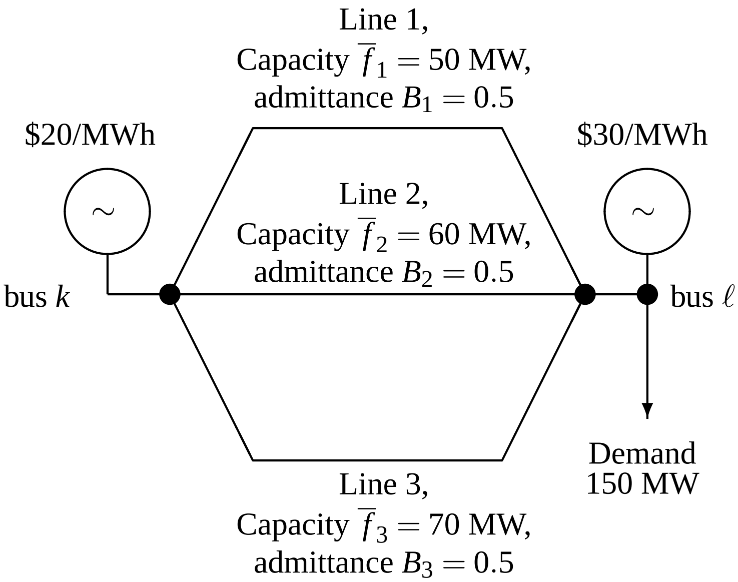

Fig. 1. Two node, three line network.

C.3 Proposed property right

We propose to define the property right for transmission to be the right to receive the revenue stream specified by (1) based on the pre-contingency flows, even when contingency constraints are binding. That is, we would suggest replacing the question marks in table I with (1) based on pre-contingency flows.

The proposed payment is easier to administer than payment based on the contingency flows, since the pre-contingency flows are measured or estimated already in typical systems. Although (1) using the pre-contingency flows is only an approximation to the more accurate payment, we will discuss in section V with reference to an example some further reasons for adopting the simpler approximate payment.

One advantage of the revenue stream that we describe is that an arbitrary collection of elements, including transmission lines, generators, and consumers, can be considered as one unit, paying the natural generalization of (1) that involves the flows at the borders between the unit and the rest of the system. This observation also motivates the name "border flow rights." The total payment to the unit is the same as the sum of the net payments to the individual elements considered separately so that the payment scheme is neutral regarding the aggregation of generation and transmission elements.

High voltage direct current transmission also fits the model we describe since it is often modelled as a paired generator and demand. This payment is consistent with the revenue stream we propose.

V. Example illustrating proposed revenue stream

We consider a simple two bus system, with busses k and ℓ, and a corridor of three lines joining the busses, lines $e = 1, 2$, and 3, each having the same admittance, with the absolute value of the imaginary part of the admittance equal to $B_e = 0.5$ units, $e = 1, 2, 3$. However, the lines have different capacities of $\overline{f}_1 = 50$ MW, $\overline{f}_2 = 60$ MW, and $\overline{f}_3 = 70$ MW, respectively. For simplicity, we assume that these capacities apply in both normal and emergency conditions. (We ignore line resistance for convenience, but this can be included in the model.) The situation is illustrated in figure 1.

There is a generator at bus k that offers its energy at \$20/MWh and a generator at bus ℓ that offers its energy at \$30/MWh. We ignore capacity constraints on the generators. There is 150 MW of demand at bus ℓ.

In section V-A, we discuss the OBSCD problem and its solution. Then in sections V-B–V-E we discuss revenue streams for remuneration of transmission lines based on, respectively, flowgate rights, marginal contribution to welfare, the approximation to marginal contribution using the pre-contingency flows, and the incremental contribution to welfare. In section V-F, we summarize and compare the remuneration schemes.

A. OBSCD problem and solution

The OBSCD problem is to minimize the accepted offer prices while meeting demand and satisfying the security constraints. (Since the demand is constant, the welfare is a constant minus the costs.) The contingency constraints require that, in the event of an outage of any transmission element, the flow on the remaining lines are within ratings. Since there are three transmission elements, there are three contingencies and a total of six contingency constraints.

Let us write q_k and q_ℓ for the generations at busses k and ℓ, respectively. The system energy balance constraint requires that $q_k + q_\ell = 150$.

We write f_e for the base-case flow on line e in the direction from k to ℓ if all lines are in service. Because of the lossless assumption, the power flow from the line into bus k is $P_{\ell k} = -f_e$, whereas the power flow from the line into bus ℓ is $P_{k\ell} = f_e$.

The non-contingency inequality constraints require that:

$$\forall e = 1,2,3, f_e \leq \overline{f}_e.$$

In the development in the appendix, we consider the base-case voltage magnitudes and phases and explicitly represent the base-case power flow equality and inequality constraints in terms of them. For this simple system, however, we can invert the power flow equality constraints and obtain the following expressions for the non-contingency inequality constraints:

$$\forall e = 1,2,3, \frac{B_e}{\sum_t B_t} q_k \leq \overline{f}_e.$$

Let us also write f_e^ω for the flow on line e if there were a contingency on line ω, for $e, \omega = 1,2,3$. Again, $-f_e^\omega$ and f_e^ω are, respectively, the contingent flow from the line into bus k and the contingent flow from the line into bus ℓ, in the event of a contingency on line ω. Naturally, $f_\omega^\omega = 0, \omega = 1,2,3$.

The contingent flows are implicit functions of q_k and q_ℓ and must be limited so that:

$$\forall e = 1,2,3, \forall \omega \neq e, f_e^\omega \leq \overline{f}_e.$$

In the appendix, we consider the contingency-case voltage magnitudes and phases and explicitly represent the contingency-case power flow equality and inequality constraints in terms of them. As with the base-case constraints, however, we can invert the contingency-case power flow equality constraints and obtain the following expressions for the contingency inequality constraints:

$$\forall e = 1,2,3, \forall \omega \neq e, \forall t \neq e, \omega, \frac{B_e}{B_e + B_t} q_k \leq \overline{f}_e.$$

For this OBSCD problem, the non-contingency inequality constraints are never binding and so we will not consider the non-contingency constraints explicitly. Moreover, because line 1 has the lowest capacity, the most binding contingency inequality constraints will always involve the flow on line $e = 1$ in the event of an outage of either line $\omega = 2$ or 3. We will not explicitly consider the four other contingency inequality constraints since they are never binding, so that in this example we can use the index of the outaged line to index the potentially binding contingency constraints. Summarizing the effects of the constraints, the secure capability to transmit from bus k to bus ℓ is 100 MW.

Omitting the constraints that are never binding, the OBSCD problem can be written as:

$$\min_{q_k, q_\ell} \left\{ 30q_k + 20q_\ell | q_k + q_\ell = 150, \frac{B_1}{B_1 + B_2} q_k \leq \overline{f}_1, \frac{B_1}{B_1 + B_3} q_k \leq \overline{f}_1 \right\}.$$

The solution of this problem is $q_k^\star = 100$ and $q_\ell^\star = 50$, with $33\frac{1}{3}$ MW flowing on each line pre-contingency and 50 MW flowing on each remaining line in the event of any contingency.[14]

The LMPs at k and ℓ are $p_k = 20$ \$/MWh and $p_\ell = 30$ \$/MWh, respectively. The two inequality constraints are identical, so that there is redundancy in the constraints: the sum of the Lagrange multipliers on these constraints is $\eta^\star = 20$ \$/MWh.

B. Flowgate rights payment

The only congested flowgate in the system is line 1, with a Lagrange multipler of $\eta^\star = 20$ \$/MWh. Using a flowgate based payment scheme based on the contingent flow on line 1, the revenue to line 1 would be:

$$\eta^\star \overline{f}_1 = 1000 \text{ \$/MWh}. \tag{2}$$

The payment to lines 2 and 3 would be zero. Note that although the payment to these lines is zero, they contribute to welfare. For example, if line 2 or 3 were to be out of service, the welfare would be reduced since the capacity to transfer cheap power to the consumer would be reduced. (In section V-E, we explicitly calculate the reduction in welfare if line 2 or 3 were out of service.)[15]

C. Payment based on contingency flows

Using the payment scheme proposed in Gribik *et al.* [1, Appendix B], the revenue stream to each line is based on the sensitivity of welfare to capacity and to admittance, which in this case is also equal to the flow on the line in the contingency case multiplied by the price difference between bus k and bus ℓ. That is, the payment is based on (1) where the flows are taken to be contingency flows. The sensitivities can be calculated from the solutions of the OBSCD problem [45], [46].

For line 1, the sensitivity of welfare to capacity is $\eta^\star$. The sensitivity of welfare to admittance is equal to $-\frac{B_2}{(B_1+B_2)^2}\eta^\star q_k^\star$. The revenue stream for line 1 is therefore:

$$\eta^\star \overline{f}_1 - \frac{B_2}{(B_1+B_2)^2}\eta^\star q_k^\star B_1 = 500 \text{ \$/h}.$$

This is equal to the revenue stream based on the LMPs and the contingent flow on line 1 (on outage of line 3):

$$p_k(-f_1^3) + p_\ell f_1^3 = 500 \text{ \$/h}.$$

For lines 2 and 3, the payment depends on the "sharing" of the Lagrange multiplier $\eta^\star$ between the two binding constraints. At one extreme, suppose that the Lagrange multiplier on the constraint $\frac{B_1}{B_1+B_2}q_k \leq \overline{f}_1$ is assumed to be equal to $\eta^\star$, while the Lagrange multiplier on the other constraint is assumed to be 0.[16] In this case the sensitivity of welfare to the capacity of line 2 is zero and the sensitivity of welfare to the admittance of line 2 is $\frac{B_1}{(B_1+B_2)^2}\eta^\star q_k^\star$. The revenue stream for line 2 is:

$$\frac{B_1}{(B_1+B_2)^2}\eta^\star q_k^\star B_2 = 500 \text{ \$/h}.$$

[14] Inverting the power flow equality constraints makes solving this OBSCD problem easy, but obscures the linear dependence of the explicit form of the constraints on the parameters $B_1, \overline{f}_1, B_2, \overline{f}_2, B_2$, and $\overline{f}_3$. The theoretical derivation in the appendix uses the explicit form of the equality and inequality constraints; however, the numerical values of the sensitivities calculated in sections V-B–V-E do not depend on the method of solution of the OBSCD problem.

[15] In some flowgate rights mechanisms, such as in ERCOT, proxy limits based on pre-contingency flows are used. For example, a pre-contingency limit of $\tilde{f}_1 = 33\frac{1}{3}$ MW might be set for line 1. A correspondingly higher Lagrange multiplier of $\tilde{\eta} = 30$ \$/MWh would then produce the same revenue stream as (2).

[16] In fact, we already adopted this assumption for calculating the revenue stream to line 1. However, for line 1, the revenue stream does not depend on how the Lagrange multiplier is shared between the two constraints.

This is equal to the revenue stream based on the LMPs and the contingent flow on line 2 (on outage of line 3):

$$p_k(-f_2^3) + p_\ell f_2^3 = 500 \text{ \$/h}.$$

The sensitivity of welfare to capacity of line 3 is zero and the sensitivity of welfare to admittance of line 3 is zero and so the revenue stream is equal to zero. This is equal to the contingent flow on line 3 on outage of line 3, which is zero, times the marginal cost price difference.

At the other extreme, the Lagrange multiplier on the constraint $\frac{B_1}{B_1+B_2} q_k \leq \overline{f}_1$ could be assumed equal to zero while the Lagrange multiplier on the other constraint would be equal to $\eta^\star$. In this case, the revenue stream to line 2 would be zero while the revenue stream to line 3 would be 500 \$/MWh.

If we "share" the Lagrange multiplier equally between the two constraints then the revenue stream for both lines 2 and 3 would be 250 \$/MWh.

D. *Payment based on pre-contingency flows*

In this section, we consider the approximate payment based on (1) and using the pre-contingency flows. The flow on each line pre-contingency is $f_e = 33\frac{1}{3}$ MW, $e = 1, 2, 3$, so that the revenue stream to each line is:

$$p_k(-f_e) + p_\ell f_e = 333\frac{1}{3} \text{ \$/h}, e = 1, 2, 3.$$

E. *Incremental payment*

We also consider payment to each line based on its incremental contribution to welfare. We can also think of this as resulting from a Vickrey auction [47] or a Groves-Ledyard mechanism [48].

If line 1 were to be removed from the system then there would be 60 MW of secure transmission capability from bus k to bus ℓ. That is, the presence of line 1 provides 40 MW of incremental capability to the system, which increases the welfare by 400 \$/h by allowing 40 MW of generation from bus k at price \$20/MWh to displace 40 MW of generation at bus ℓ at price \$30/MWh. The payment to line 1 would be 400 \$/h under this payment scheme.

Similarly, lines 2 and 3 each provide 50 MW of incremental capability, with a corresponding contribution to welfare of 500 \$/h. The payment to lines 2 and 3 would be 500 \$/h. Lines 2 and 3 provide more incremental capability to the system than line 1.

The sum of the three incremental payments exceed the total congestion rental for the system of 1000 \$/h. That is, using the incremental payments would violate the ISO revenue adequacy requirement. One approach to this issue is to scale the payments down so that they exactly match the congestion rental. Scaling all payments by the same factor to make the payments revenue neutral would yields payments of 285.7 \$/h, 357.1 \$/h, and 357.1 \$/h, respectively.

F. *Summary and comparison*

Table II shows the revenue streams under the various payment schemes. For the payment scheme based on (1) and using the contingent flows, we have chosen to share the revenues equally between lines 2 and 3.

To consider the incentives provided by each payment scheme, first consider the options typically open to a builder of a new transmission line. Such a builder is faced with building a relatively large, lumpy contribution to the system. The efficient incentive for such a decision is the incremental change to welfare. Unfortunately, paying all lines the incremental change to welfare is not revenue adequate if, for example, the system is contingency constrained, and presents a revenue adequacy problem for the ISO. Since transmission systems are generally contingency constrained, we can expect that in general there will be a revenue shortfall with such a payment scheme, even in the absence of economies of scale in transmission construction costs.[17]

[17] Revenue inadequacy would be avoided if the incremental payment were made only to new transmission construction. However, this institutionalizes a permanent asymmetry between payment for old and new transmission.

TABLE II
REVENUE STREAMS, IN $/H, UNDER VARIOUS PAYMENT SCHEMES.

Line	1	2	3
Flowgate	1000	0	0
Based on (1) and contingency flows	500	250	250
Based on (1) and pre-contingency flows	$333\frac{1}{3}$	$333\frac{1}{3}$	$333\frac{1}{3}$
Incremental	400	500	500
Scaled incremental	285.7	357.1	357.1

A scaled version of incremental welfare might be used to enforce revenue adequacy; however, the incentive provided is no longer as closely related to the incremental contribution to welfare. In summary, payment schemes based directly on incremental change to welfare are problematic.

We consider payment based on flowgate rights. Under flowgate rights, lines 2 and 3 are indifferent to being in-service or out-of-service with such a flowgate rights payment scheme. This is again problematic.[18]

To analyze the payment schemes based on (1) consider the options open to an owner of an existing line to *change* the electrical properties of the line. Reconductoring and series capacitors can change the thermal capacity and the admittance, respectively.[19] Moreover, there are various reconductoring options and various values of series capacitors that could be used to achieve a fine grained change in the thermal capacity or the admittance. Although such options for changing the electrical characteristics of a line are likely to have economies of scale, the total change in transmission capability may be a relatively small fraction of the existing capability in a corridor. Consequently, a marginal signal is likely to be useful in these cases.

Examination of table II shows that using (1) based on pre-contingency flows yields a payment that is closer to the incremental change in welfare for this case. For lines 2 and 3, it yields a revenue stream that is intermediate between:

- the revenue specified by (1) using contingency flows and
- the incremental contribution as calculated in a Vickrey auction [47] or by a Groves-Ledyard mechanism [48].

It would be interesting to test empirically whether (1) based on the pre-contingency flows is a workable approximation to (1) based on the contingency flows when averaged over various operating conditions. We hypothesize that it might be. As mentioned in section IV-C, it would be easier to administer a payment based on pre-contingency flows than one based on contingency flows. In the discussion in sections VI and following, either the exact payment scheme based on contingency flows or the approximation based on pre-contingency flows could be used.

VI. CONTRACTS FOR DIFFERENCES OF DIFFERENCES

The proposal (1) for remuneration of a transmission owner, whether based on pre-contingency or contingency flows, is independent of any financial contract definition. In this section, we describe how the revenue stream can be used to fund a risk hedging financial instrument for transmission customers. In section VI-A, we provide a description of the basic mechanism, while in section VI-B we provide an interpretation in terms of contract paths. Finally, in section VI-C, we consider option rights.

A. Description

First consider a generator and a consumer at a bus ℓ. Suppose that the generator and consumer have signed a CFD to hedge the LMP at bus ℓ. The CFD provides a side payment from consumer to generator equal to a

[18] In principle, we could consider all three lines together as a unit and share the net proceeds, by redistributing the congestion rental. Border flow rights does precisely this.

[19] In many urban areas with limited corridor space, such upgrades may be the only options. See [49] for a summary of estimated costs.

contract quantity q times the difference between a strike price for energy and the LMP at bus ℓ. The variation of LMP at ℓ is hedged for production by the generator equal to q and also hedged for consumer demand equal to q, since the sum of the LMP payments and the payment due to the CFD is always equal to q times the strike price. The CFD allows the generator and consumer, who have equal and opposite exposures to the variation of LMP at a bus, to both costlessly hedge their price risks for the quantity q.

Now suppose that the generator is located at a different bus, bus k. We now assume that the generator at k and the consumer at ℓ have signed a CFD to hedge the LMP at bus k for a contract quantity q. As is well known, a CFD based on the LMP at bus k is insufficient to hedge the price differences between busses k and ℓ. In a market with point-to-point FTRs, the consumer could hedge the price difference by purchasing an appropriate FTR of quantity q with point of injection k and point of withdrawal ℓ.

In contrast to an FTR-only system, we propose an alternative hedging instrument here, called a "contract for differences of differences" (CFDD). The CFDD is a contract between:

- the consumer (or generator or both) and
- the owner of the transmission line or lines joining bus k and ℓ.

The CFDD provides for a side payment to the transmission line owner equal to a contract quantity times the difference between:

- a strike price for transmission services and
- the *difference* between the LMPs at bus k and ℓ.

The CFDD is so-called because it pays based on the *difference* between a strike price and LMP *differences* between two busses. Given a contract quantity of q, the CFDD would hedge the variation in LMP differences since the net payment for transmission services would always be q times the strike price.

A transmission owner and its transmission customer have equal and opposite exposures to the variation of LMP differences. As with the CFD, the CFDD allows them both to hedge their price risk for the quantity q. That is, in addition to defining an underlying revenue stream in table I by (1), we would also suggest replacing FTRs by CFDDs. With these changes to the transmission property rights definition, there is a symmetry between energy and transmission markets: just as generators and demand have opposite exposures to risks of nodal price variation that can be hedged with CFDs, transmission owners and customers have opposite exposure to risks of nodal price *difference* variation that can be hedged with CFDDs.

In the case of an FTR-only system, revenue adequacy for the ISO requires the FTRs to satisfy the SFT test. In contrast, with the proposed transmission property right and the use of CFDDs, revenue adequacy associated with issuing financial transmission rights is devolved to transmission line owners. If transmission owners over-sell compared to their actual flows then they are responsible for the shortfall. If they under-sell compared to their actual flows then they will receive a volatile revenue stream.

Furthermore, during a transmission outage, the transmission owner will still be liable for the CFDD payment, providing a powerful incentive to the transmission line owner to make the line available and in-service whenever congestion in the network makes LMP differences high.[20] The risk of ill-timed transmission outages is transferred from the ISO and transmission customers, who have little fundamental control of transmission outages, to the transmission owners themselves, who have more control and an ability to focus on long-term reliability issues.[21]

To summarize, since the CFDD is a purely financial contract there is no need for any derating policy administered by the ISO. The transmission owner is financially responsible for its outages. The OBSCD can take place independently of financial positions of transmission owners and the purchasers of transmission rights. As in the case of CFDs, however, prudential requirements may be appropriate for transmission owners that take on more risk than is covered by likely anticipated flows. Moreover, trading of CFDDs can also be facilitated by an exchange. To calculate the amount of CFDD payment that is covered by the revenue stream, a version of the simultaneous feasibility test can be conducted as will be discussed in section VII-C.

[20]While a CFDD could have provisions to become inactive when a transmission line is out of service, this would reduce the hedging value. We assume that CFDs and CFDDs have no provisions to be inactive during outage conditions.

[21]This issue is particularly problematic for FTR and flowgate mechanisms, since lines other than the binding constraints can reduce transfer capability if out of service, but such lines receive no payment.

B. Contract paths

In the example in figure 1, busses k and ℓ are directly connected by transmission. In general, a generator may want to hedge a transaction of quantity q against variation in the LMP differences between busses k and ℓ that are not directly connected by a transmission line. In this case, a sequence of CFDDs can be assembled from bus k through intermediate lines to bus ℓ. A CFDD is purchased on each intermediate line (or collection of lines) for quantity q, just as in the contract path paradigm. The sum of the revenue streams from the CFDDs will exactly hedge the transaction from bus k to bus ℓ. That is, a "contract path" of CFDDs can be used to hedge the LMP difference.

Moreover, if there are multiple parallel paths from bus k to bus ℓ then any one of the contract paths, or an appropriate combination of several of the contract paths, can be used to hedge the LMP difference. This means that different paths can effectively compete to supply CFDDs to transmission customers.

Although this mechanism is reminiscent of the contract path paradigm for transmission services as traditionally implemented by North American utilities, it is important to emphasize that the CFDD mechanism respects Kirchhoff's laws and security constraints, since physical flows on the lines are always determined by the ISO as the result of a offer-based, security-constrained dispatch. Unlike the contract path paradigm, which does not support *bona fide* competition in transmission provisions, the CFDD mechanism does support competition between sellers of CFDDs on parallel paths.

C. Option rights

The CFDDs described so far have been so-called "obligation" rights, which have negative payoff if the sign of the LMP difference reverses. However, the financial nature of these rights provides significant flexibility to define "option" rights that have zero payoff when the sign of an LMP difference reverses. In fact, the revenue stream (1) to a line will usually be positive, irrespective of the sign of the LMP difference, since the direction of flow is usually reversed when the LMP price difference is reversed.

Consequently, the line owner can sign an option CFDD for flow in each direction. This matches the underlying physical capability of the line, which allows flow in either direction and contrasts with the straightforward and most usual definition of an FTR. The drawback of FTRs, described in section IV-A, of requiring a nomination that applies throughout an FTR auction contract period, is not problematic for the border flow right and for the associated financial right of CFDDs.

Nevertheless, there are cases where the border flow right would produce a negative revenue stream. In particular, if the marginal contribution to welfare of the line is negative then the revenue stream may be negative.[22] The line can avoid such a negative revenue stream by disconnecting itself from the network and can usually be expected to make an incremental improvement in welfare by doing so.[23]

VII. Trading of CFDDs

The CFDD mechanism allows for trading of financial transmission rights by entities other than the ISO because the simultaneous feasibility test to protect the revenue adequacy of the ISO is not part of the CFDD mechanism. In principle, the trading can even be completely decentralized. Decentralized trading will be discussed in section VII-A.

Nevertheless, it may be helpful for transmission customers to be able to purchase contract paths of CFDDs that are assembled through a centralized exchange and for transmission owners to sell their CFDDs through such an exchange. There are at least two ways that this could be done, as will be described in sections VII-B

[22]Because (1) based on the pre-contingency flows is only an approximation to the marginal value of the line, the revenue stream could be positive even if the marginal contribution to welfare were negative, and vice versa.

[23]See [36, §5.1 and Figure 5] for an example. A line in that example is not at capacity, but the sensitivity of welfare to its admittance is negative. Welfare could presumably be improved if the line were to disconnect from the system. However, this is not always the case and the ISO should have the right to "commit" a line that increases welfare by being connected. The ISO could compensate the line in the same manner that "make-whole" payments are provided to generators that are committed but do not cover their offer costs from LMP payments [50]. Conversely, we will see in the discussion of merchant construction in section VIII that we should allow the ISO to "de-commit" a line if its being in-service decreases welfare.

and VII-C. In either case, there is no need for the ISO to be involved in the exchange. That is, the exchange can focus on the longer-term issues of transmission rights, while the ISO remains focused on day-to-day issues. Moreover, such an exchange could trade CFDs and CFDDs simultaneously, providing long-term financial hedging instruments that would facilitate both generation and transmission capital formation.

We will describe each of these three alternatives for trading and provide examples to illustrate, basing payments on (1) using pre-contingency flows. The trading alternatives are then summarized in section VII-D.

A. Decentralized trading

A.1 Description

No central reconfiguration or intervention of the CFDDs are necessarily required, so long as transmission owners can bear the risk that payments to transmission customers may exceed the revenue from the OBSCD if CFDDs are "over-sold." Furthermore, portions of the capacity can be traded for differing durations to support both long-term and short-term hedging needs.

A.2 Example with fixed demand

Consider the system shown in figure 1, with demand at bus ℓ of 150 MW. Suppose that the three lines were owned by different entities (or, more generally, that each line stands for a transmission system owned by a different entity, with all three systems interconnected at bus k and also interconnected at bus ℓ.)

From the analysis in section V, the flow on each line is $33\frac{1}{3}$ MW when flow between bus k and bus ℓ is at its security-constrained maximum. Because of the purely financial nature of CFDDs, any entity could offer to sell any quantity of CFDD. However, we will not consider speculative trades and assume that each line owner offers to sell $33\frac{1}{3}$ MW of CFDD. If the demand at ℓ were seeking to hedge price volatility on all of its demand of 150 MW then it would seek to purchase all 100 MW of offered CFDDs. Together with a CFD for 100 MW with the generator at bus k, the CFDD would completely hedge the volatility of the price of importing the 100 MW. (A CFD for 50 MW with the generator at bus ℓ would hedge its remaining demand.)

A.3 Example with on- and off-peak demand

Suppose that in the example of figure 1, the demand were 150 MW on-peak and 50 MW off-peak. CFDDs can, in principle, be offered both on- and off-peak. If both on- and off-peak CFDDs are available then the demand might not purchase CFDDs for off-peak delivery but only purchase 100 MW of CFDDs for on-peak delivery.

A.4 Discussion

Decentralized trading may be adequate for systems with few participants, but is likely to be unwieldy when there are multiple participants desiring to hedge transmission. In the next section, we will consider the first of two centralized trading alternatives.

B. Network flow based clearing

B.1 Description

Suppose that individual line owners specify offers for strike prices and quantities of CFDDs referenced between the ends of their lines. If transmission customers also bid their willingness to pay to purchase contract paths of CFDDs between injection and withdrawal points then a simple "network flow" based algorithm can be used to optimally match the bids and offers [51].

B.2 Example

Suppose that lines $e = 1, 2, 3$ each offer $33\frac{1}{3}$ MW of CFDDs at prices of, respectively, \$1/MWh, \$2/MWh, and \$3/MWh, for all hours of a contract period. Suppose that the consumer at bus ℓ bids for 150 MW of

TABLE III
REVENUE STREAMS FOR 50 MW DEMAND.

Revenue stream description	Amount
Consumer at ℓ pays ISO at LMP	$50 \times 20 = \$1000$/h
consumer at ℓ pays transmission owner e based on CFDD	$33\frac{1}{3} \times (4-(0-0)) = \$133\frac{1}{3}$/h
ISO pays transmission owner e at LMP difference	$16\frac{2}{3} \times (0-0) = \$0$/h
ISO pays generator at bus k at LMP	$50 \times 20 = \$1000$/h
ISO pays generator at bus ℓ at LMP	$0 \times 20 = \$0$/h

TABLE IV
REVENUE STREAMS FOR 150 MW DEMAND.

Revenue stream description	Amount
Consumer at ℓ pays ISO at LMP	$150 \times 30 = \$4500$/h
Consumer at ℓ pays transmission owner e based on CFDD	$33\frac{1}{3} \times (4-(30-20)) = -\$200$/h
ISO pays transmission owner e at LMP difference	$33\frac{1}{3} \times (30-20) = \$333\frac{1}{3}$/h
ISO pays generator at bus k at LMP	$100 \times 20 = \$2000$/h
ISO pays generator at bus ℓ at LMP	$50 \times 30 = \$1500$/h

CFDDs at \$4/MWh for all hours of a contract period. All 100 MW of offered CFDDs are sold and the clearing strike price is \$4/MWh.

We now consider the revenue stream under OBSCD under two demand conditions and assuming that the generator at bus k offers energy at \$20/MWh and the generator at bus ℓ offers energy at \$30/MWh. We initially assume that there are no CFDs.

Demand of 50 MW. In this case, OBSCD yields 50 MW of generation at bus k, 0 MW of generation at bus ℓ, and LMPs of $p_k = p_\ell =$ \$20/MWh with no transmission congestion. The revenue streams are shown in table III. By construction, the ISO net income is zero. The consumer net payment is \$1400/h or an average price, including transmission payments, of \$28/MWh. On net, each transmission owner e is paid $\$133\frac{1}{3}$/h. The generator at bus k is paid \$1000/h, while the generator at bus ℓ is paid nothing.

Demand of 150 MW. In this case, OBSCD yields 100 MW of generation at bus k, 50 MW of generation at bus ℓ, and LMPs of $p_k =$ \$20/MWh and $p_\ell =$ \$30/MWh with transmission congestion. The revenue streams are shown in table IV. By construction, the ISO net income is again zero. The consumer net payment is \$3900/h or an average price, including transmission payments, of \$26/MWh.[24] On net, each transmission owner e is paid $\$133\frac{1}{3}$/h. The generator at bus k is paid \$2000/h, while the generator at bus ℓ is paid \$1500/h.

B.3 Discussion

In the previous example, the revenue stream to each transmission owner e is constant irrespective of demand level, indicating that transmission owner e has hedged the risk of volatile LMP difference payments by selling CFDDs. This is a powerful incentive for transmission owners to sell CFDDs. The strike price, together with the distribution of the demand, will determine the financial position of the transmission owner relative to the expected revenue from the unhedged revenue stream (1).

The revenue stream paid by the consumer at ℓ is more constant than it would be in the absence of the

[24]The average price is lower than for the off-peak demand of 50 MW because we have included the transmission payments on a per MWh basis. Note, however, that some of the transmission payments are effectively sunk costs at the time of dispatch so that the marginal cost is higher on-peak than off-peak.

CFDDs. Without CFDDs, the consumer would pay \$20/MWh in off-peak and \$30/MWh on-peak. Hedging of transmission risk is a powerful incentive for the consumer to purchase CFDDs.

Moreover, as discussed in section VI, transmission customers and transmission owners have opposite tastes for exposure to LMP difference fluctuations and they both hedge their risk by signing CFDDs. Analogously to CFDs, transmission customers and transmission owners can both costlessly hedge their risk by signing CFDDs.

Furthermore, the consumer could also hedge is exposure to energy price variation by signing CFDs with the generators at busses k and ℓ. The clearing auction could easily be expanded to include offers for generation as well as offers for transmission service.

However, a drawback of the network flow solution is that it is unlikely to correspond to an actual dispatch for any given line except for very simple networks. Consequently, the revenue streams due to CFDDs may not match the payment from (1), even on average. In the next section, we discuss an alternative trading mechanism where the match is likely to be closer.

C. Simultaneous feasibility test clearing

C.1 Description

Instead of clearing based on the network flow solution, line owners may want to sign CFDDs only up to what they are sure will be the actual flow on their lines in contingency-constrained conditions. In this case, a modified OBSCD similar to an FTR auction would enable CFDDs to be assembled. Transmission owners would effectively be offering their capacity, possibly as "price takers," and the prices would be set by the transmission customer bids.

The solution of the auction specifies the CFDD contract quantity and strike price for each transmission element. The transmission owner is then paid the contract quantity times the strike price, with the contract quantity times the LMP differences from the OBSCD refunded by the transmission owner to its transmission customers. While this scheme is collectively revenue adequate assuming all lines are in service (using the same argument that shows that the ISO is revenue adequate under conventional FTRs), individual lines may be revenue inadequate if the flows and binding constraints under actual dispatch differ significantly from the solution of the auction or if lines are out of service. Mitigation of the risk of revenue inadequacy for individual lines could be accomplished through ownership of several lines, similarly to the way in which ownership of a portfolio of generators mitigates the risk that revenue is inadequate for individual generators when out-of-service.

C.2 Example with a single bid for CFDDs

We again consider the system of figure 1. Assume that each transmission owner e offers its capacity as a price taker. Also suppose that the consumer at ℓ bids for 150 MW of CFDDs from k to ℓ at \$4/MWh for all hours in a contract period. Performing OBSCD, there would again be 100 MW of CFDD at a strike price of \$4/MWh.

C.3 Example with multiple bids for CFDDs

If there are multiple bids for CFDDs and multiple owners of transmission then the allocation of contracts to transmission owners is not unique. To see this, suppose that in the system of figure 1 there are two successful bidders for the 100 MW of CFDDs between k and ℓ and that each bidder receives 50 MW of CFFDs. The assignment of contracts between lines $e = 1,2,3$ and bidders is arbitrary. In particular, one of the bidders might sign CFDDs for 30 MW with line 1 and for 20 MW with line 2, or it could sign any other combination adding up to 50 MW.

A natural way to resolve this arbitrariness in a DC power flow model would be to assign contracts based on power transfer distribution factors, so that in this case each bidder would receive $16\frac{2}{3}$ MW of CFDD from each line.

C.4 Example with previously allocated capacity

Suppose that 50 MW of CFDDs from k to ℓ had already been signed. For concreteness, suppose that there were $16\frac{2}{3}$ MW of previously allocated CFDDs with each of the lines $e = 1,2,3$. These CFDDs might have been arranged in a previous auction or as part of a long-term contract signed outside the centralized exchange.

An auction for the remaining capacity can be performed by modeling the previously allocated CFDDs as a 50 MW injection at k and 50 MW withdrawal at ℓ. The new bids could then be cleared as previously.

C.5 Example with new capacity offer

Suppose there was a proposal for construction of a fourth transmission line in parallel with the three existing lines in figure 1. Suppose that 40 MW of transmission capacity was offered at a price of $3/MWh, with the new line having the same admittance as the three other lines. The additional line will increase the capability from k to ℓ to 120 MW. Assuming that no CFDDs had been signed on the system previously, and that the consumer at ℓ again bids for 150 MW of CFDDs from k to ℓ at a price of $4/MWh, then 120 MW of CFDDs are cleared at a strike price of $4/MWh.[25]

C.6 Example with energy offers and bids

We now consider an auction involving both forward transmission and forward energy offers and bids in the system in figure 1. Suppose that energy was offered by the generator at bus k at 20 $/MWh and by the generator at bus ℓ at 30 $/MWh for all hours in a contract period. Furthermore, suppose that the consumer at bus ℓ bid for 50 MW of delivered power at bus ℓ at 28 $/MWh and for 100 MW of transmission service at 4 $/MWh. Furthermore, suppose all lines offer in their capacity as price takers. In this case, 50 MW of generated power is cleared at bus k at 20 $/MWh, 50 MW of delivered power is cleared at bus ℓ at 24 $/MWh, and 50 MW of transmission service is cleared at 4 $/MWh. This establishes forward positions for both energy and transmission.

D. Summary

The discussion in section VII-A shows that the CFDDs can be traded in a decentralized fashion, albeit with risk to the transmission owner if the contract quantity exceeds the actual flow on the line when the LMP price difference equals or is above the strike price. This risk may be acceptable in well-established systems with predictable flows when there is congestion; however, trading through an exchange can be arranged to reduce the risk.

In both proposals for centralized trading of transmission rights, the auctions arrange for financial hedges to transmission customers, but without necessitating the participation of the ISO. The actual dispatch is determined by the OBSCD, with no reference to the strike price of the CFFDs.[26]

VIII. MERCHANT CONSTRUCTION

Merchant generators hoping to build new construction can be expected to desire to sign long-term contracts (both CFDs and CFDDs) to hedge themselves against LMP variation and to lock in prices in advance that their presence in the spot market may depress. Liquid forward long-term energy markets are an important part of encouraging merchant generation investment.

Similarly, merchant transmission providers can utilize forward markets to enable them to sign long-term contracts to hedge LMP variation and also lock in LMP differences. Because of lumpiness and economies of scale in transmission construction, it is conceivable that a transmission addition will significantly reduce LMP

[25] In general, we need to explicitly consider the decision of whether or not the offered line is built by incorporating an integer variable into the auction formulation. In this example, however, we can determine that the new line should be built. Note that the pre-existing transmission will receive a different revenue stream once the new transmission is built. There are two reasons for this change: the flow on each of the pre-existing lines will change and the LMP differences will be no greater.

[26] In the case of merchant transmission, it may be appropriate to allow a non-zero price for transmission offered into the OBSCD and possibly into the CFDD auction. See O'Neill *et al.* [50] for related discussion.

differences between the ends of the line. The ability to sign contracts that are based on forward nodal energy price differences would allow such merchant transmission investment to be profitable despite depressing the LMP difference.

To date, only merchant high-voltage direct current (DC) transmission, which acts as a simultaneous buyer and seller of energy and receives a revenue stream given by (1), has appeared in any markets in the world. The introduction of border flow rights as a property right definition for *all* transmission would allow merchant AC transmission to also function as a simultaneous buyer and seller of energy, as implied by the revenue stream (1). The combination of border flow rights and CFDDs would help to enable the financing of merchant AC transmission through forward contracts for energy. To summarize, CFDDs, built on border flow rights, allow forward energy and transmission markets to support the development of merchant AC and DC transmission.

The value of transmission in providing for flow in both directions is compensated directly under border flow rights. This feature of border flow rights avoids a drawback of current FTR formulations where deviation between the nomination of the FTR and the actual patterns of dispatch can reduce or negate the FTR payment. Moreover, there is considerable flexibility for a transmission provider to offer option CFDDs and other financial products that considerably generalize point-to-point obligation FTRs.

As mentioned in the introduction, Bushnell and Stoft show that, under somewhat restrictive assumptions, any transmission investment that is detrimental to the grid will result in FTRs that have negative value [11], [12], [13]. Such a result does not extend to our proposed revenue stream as defined in (1). For example, consider again the construction of a fourth line in parallel with the three lines shown in figure 1. If the line had the same admittance as the others, but had a capacity of only 10 MW then the welfare would be reduced by the presence of this line, even though it would be paid a positive amount under (1). A natural solution to this issue is presented in [50], where the ISO has flexibility to "commit" or "de-commit" transmission lines in a way that is analogous to commitment of generation. In particular, if the ISO has flexibility to disconnect a line if its presence reduces welfare, then the 10 MW line would be disconnected any time that the demand at bus ℓ exceeded 30 MW.[27]

Moreover, the benefit due to an expansion of the capacity of line 1 will accrue to all three lines. Indeed, by design, the marginal benefit of added capacity of line 1 is shared amongst all three lines. Consequently, even in the absence of lumpiness, an efficient level of investment requires financing by coalitions of beneficiaries. While there is somewhat cumbersome, transmission expansion currently involves such mechanisms in many jurisdictions.

IX. Conclusion

In this paper, we have proposed a property right for transmission based on the approach of Gribik *et al.* [2], by defining an underlying revenue stream that accrues to the owner of a transmission line. Under the proposed border flow rights model, the owner of a transmission line is paid at the locational marginal price for net energy that it delivers to the rest of the system. Border flow rights provide an approximation to efficient marginal incentives for transmission expansion by coalitions of beneficiaries.

Based on the property right for transmission, we have proposed a financial right for hedging LMP differences, called a contract for differences of differences, and provided examples of its use. The CFDD is based on the underlying revenue stream in the border flow right. Unlike previous FTR formulations, we first define a property right in terms of an underlying revenue stream that is independent of FTR nominations and then define a financial right that is built on the underlying property right.

Analogously to contracts for differences, contracts for differences of differences can be traded without an ISO. Nevertheless, exchange trading of CFDDs has several advantages over completely decentralized trading. Furthermore, both transmission and energy can be traded forward in one exchange, avoiding the bifurcation

[27] Anecdotally, it appears that some lower voltage lines in the North American electricity system in fact lower welfare when they are in-service because their capacity determines a binding contingency constraint. We have not explored the full implications of ISO flexibility to remove all such lines from service.

in current markets. The ISO could be, but does not have to be, involved in the exchange.

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Appendix: Equivalence of sensitivity based payment and (1)

We formulate the transmission constrained welfare maximization problem. Our formulation explicitly represents line and other transmission element parameters and limits that appear *linearly* in the power flow equality and inequality constraints when the base-case and contingency-case constraints are represented explicitly. We illustrate the formulation with reference to the example problem shown in figure 1.

Our notation is, in part, based on that in [7]. In particular, let $y \in \mathbb{R}^{n_y}$ be the vector of net power loads at all the busses and let $b : \mathbb{R}^{n_y} \to \mathbb{R}$ be the net benefits of consumption minus costs of production at the busses. We assume that there may also be a vector of controls $u \in \mathbb{R}^{n_u}$ that the ISO can adjust, potentially within limits. (Depending on the formulation, y may include both real and reactive power net loads, or just real power net power loads, or might be expanded to include any collection of quantities that are conserved nodally, including reserves. In the following, we will refer to y as the "net loads," with the understanding that it might encompass more than just real power.)

We differ from the formulation in [7] in some respects. In particular, we will explicitly represent voltage magnitudes $V \in \mathbb{R}^{n_V}$ and phases $\theta \in \mathbb{R}^{n_\theta}$ at all the busses (including the slack bus). We also consider a vector, $\chi \in \mathbb{R}^{n_\chi}$, of *parameters* and *limits* of transmission system elements.

In section A, we first consider the case ignoring contingency constraints and then incorporate contingency constraints in section B.

A. No contingency constraints

In this section, we consider the case where only constraints on base-case operations are modelled.

A.1 Net loads, objective, and other variables for example system

The generation in the example in figure 1 would appear in the vector y as negative net loads, $-q_k$ and $-q_\ell$, respectively. In particular, $y \in \mathbb{R}^2$, with:

$$y = \begin{bmatrix} -q_k \\ -q_\ell + 150 \end{bmatrix},$$

and $b : \mathbb{R}^2 \to \mathbb{R}$ with:

$$\forall y \in \mathbb{R}^2, b(y) = -20y_1 - 30(150 - y_2).$$

If we use a DC power flow, we can ignore the vector V. There are no controls u. There is an angle for each bus, so that $\theta = \begin{bmatrix} \theta_k \\ \theta_\ell \end{bmatrix} \in \mathbb{R}^2$.

A.2 Constraints

In addition to the objective of the welfare maximization problem, there are equality and inequality constraints on base-case flows. The equality constraints arise from Kirchhoff's laws and relate the vector of net loads to the voltage magnitudes, voltage phases, controls, and parameters and limits. We assume a very general form for this functional relationship. In particular, we assume that there is a matrix valued function $S : \mathbb{R}^{n_V} \times \mathbb{R}^{n_\theta} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_y} \times \mathbb{R}^{n_\chi}$, such that the net loads satisfy:

$$y = S(V, \theta, u)\chi. \tag{3}$$

That is, the parameters χ enter linearly into the specification of the power flow equations, while the effect of voltage magnitudes, voltage phases, and controls is, in principle, arbitrary.

For example, consider a transmission line e, with parameters that are specified by a sub-vector $\chi_e = \begin{bmatrix} G_e \\ B_e \\ \overline{f}_e \end{bmatrix}$ of χ. The parameters G_e, B_e, and $\overline{f}_e$ are, respectively, the absolute value of the real and imaginary parts of the series admittance of the line and the capacity of the line. (We will discuss $\overline{f}_e$ when we define the inequality constraints.)

Suppose that the element e joins busses k and ℓ and that the k-th and the ℓ-th entries of y specify real power balance at busses k and ℓ respectively. For concreteness, partition χ into $\chi = \begin{bmatrix} \chi_e \\ \chi_R \end{bmatrix}$, where χ_R specifies the parameters of the rest of the system. Similarly, partition S into $S = \begin{bmatrix} S_e & S_R \end{bmatrix}$ and let S_{ke} and $S_{\ell e}$ be the k-th and ℓ-th rows of S_e, respectively.

Then, using the AC power flow formulation,

$$\begin{aligned} S_{ke} &= \begin{bmatrix} -V_\ell V_k \cos(\theta_k - \theta_\ell) & -V_\ell V_k \sin(\theta_k - \theta_\ell) & 0 \end{bmatrix}, \\ S_{\ell e} &= \begin{bmatrix} -V_\ell V_k \cos(\theta_\ell - \theta_k) & -V_\ell V_k \sin(\theta_\ell - \theta_k) & 0 \end{bmatrix}, \end{aligned}$$

with all of the other rows of S_e equal to the zero function, so that:

$$\begin{aligned} S_{ke}\chi_e &= -V_\ell V_k [G_e \cos(\theta_k - \theta_\ell) + B_e \sin(\theta_k - \theta_\ell)], \\ S_{\ell e}\chi_e &= -V_\ell V_k [G_e \cos(\theta_\ell - \theta_k) + B_e \sin(\theta_\ell - \theta_k)]. \end{aligned}$$

Therefore, (3) reproduces the appropriate terms in the real power flow balance equations, since power balance at busses k and ℓ requires that:

$$\begin{aligned} y_k &= -V_\ell V_k [G_e \cos(\theta_k - \theta_\ell) + B_e \sin(\theta_k - \theta_\ell)], \\ y_\ell &= -V_\ell V_k [G_e \cos(\theta_\ell - \theta_k) + B_e \sin(\theta_\ell - \theta_k)]. \end{aligned}$$

We assume that there is also a matrix valued function $F : \mathbb{R}^{n_V} \times \mathbb{R}^{n_\theta} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_F} \times \mathbb{R}^{n_\chi}$ such that:

$$F(V,\theta,u)\chi \leq \mathbf{0}, \tag{4}$$

specifies the inequality constraints in the system. Moreover, we assume that F is block diagonal, with each diagonal block specifying inequality constraints relating to a different transmission element. In particular, using the example transmission element e above, we assume that F partitions into:

$$F = \begin{bmatrix} F_e & \mathbf{0} \\ \mathbf{0} & F_R \end{bmatrix},$$

where F_e specifies the constraints for element e and F_R specifies the constraints for the rest of the system. Given a real power limit of $\overline{f}_e$ on this line, F_e could be specified as:

$$F_e = \begin{bmatrix} V_\ell V_k \cos(\theta_k - \theta_\ell) & V_\ell V_k \sin(\theta_k - \theta_\ell) & -1 \\ V_\ell V_k \cos(\theta_\ell - \theta_k) & V_\ell V_k \sin(\theta_\ell - \theta_k) & -1 \end{bmatrix},$$

with flow constraints on e then given by $F_e\chi_e \leq \mathbf{0}$. That is:

$$\begin{aligned} V_\ell V_k[G_e \cos(\theta_k - \theta_\ell) + B_e \sin(\theta_k - \theta_\ell)] - \overline{f}_e &\leq 0, \\ V_\ell V_k[G_e \cos(\theta_\ell - \theta_k) + B_e \sin(\theta_\ell - \theta_k)] - \overline{f}_e &\leq 0. \end{aligned}$$

Other constraints can be represented similarly.[28]

A.3 Constraints for example system

The specification of:

$$\begin{aligned} \chi &= \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{bmatrix} \in \mathbb{R}^6, \\ S &= \begin{bmatrix} S_1 & S_2 & S_3 \end{bmatrix} : \mathbb{R}^2 \to \mathbb{R}^{2\times 6}, \text{ and,} \\ F &= \begin{bmatrix} F_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & F_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_3 \end{bmatrix} : \mathbb{R}^2 \to \mathbb{R}^{3\times 6}, \end{aligned}$$

for the example system, using the DC power flow formulation, are:

$$\begin{aligned} \chi_e &= \begin{bmatrix} B_e \\ \overline{f}_e \end{bmatrix} \in \mathbb{R}^2, e = 1,2,3, \\ \forall\theta \in \mathbb{R}^2, S_e(\theta) &= \begin{bmatrix} -(\theta_k - \theta_\ell) & 0 \\ -(\theta_\ell - \theta_k) & 0 \end{bmatrix}, e = 1,2,3, \text{ and,} \\ \forall\theta \in \mathbb{R}^2, F_e(\theta) &= \begin{bmatrix} (\theta_k - \theta_\ell) & -1 \end{bmatrix}, e = 1,2,3. \end{aligned}$$

A.4 Welfare optimization problem

To summarize, the welfare optimization problem is formulated as:

$$\max_{y\in\mathbb{R}^{n_y}, V\in\mathbb{R}^{n_V}, \theta\in\mathbb{R}^{n_\theta}, u\in\mathbb{R}^{n_u}} \{b(y) | y = S(V,\theta,u)\chi, F(V,\theta,u)\chi \leq \mathbf{0}\},$$

which we assume has a solution $y^\star, V^\star, \theta^\star, u^\star$. This formulation differs from (4) of [7] in that we have explicitly kept the dependence of the constraints on voltage magnitudes and phases. The reason for doing this is to maintain a form for the constraint functions that is linear in χ. This will enable us to prove the main result very directly.

[28] For example, a current limit can be parametrized in this way if the magnitude of the admittance is included as a parameter.

A.5 Lagrangian

The Lagrangian of the welfare problem is:

$$\mathcal{L}(y,V,\theta,u,p,\eta) = b(y) - p^\dagger(y - S(V,\theta,u)\chi) - \eta^\dagger F(V,\theta,u)\chi,$$

where p and η are dual variables and superscript $\dagger$ means transpose. Let $p^\star$ and $\eta^\star$ be the vectors of Lagrange multipliers corresponding to the solution $y^\star, V^\star, \theta^\star, u^\star$. We note that $p^\star$ is the vector of nodal prices and that we can partition $\eta^\star$ into $\eta^\star = \begin{bmatrix} \eta_e^\star \\ \eta_R^\star \end{bmatrix}$, where $\eta_e^\star$ are the Lagrange multipliers corresponding to constraints on operation of element e.

A.6 Revenue stream based on border flow right and nodal prices

We note that $S_e(V^\star,\theta^\star,u^\star)\chi_e$ represents the net border flows from element e into the rest of the system. Paying for that border flow at nodal prices results in a revenue stream of $[p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e$ to element e. This is the general case of (1). Summing this expression over all elements e yields the total payments to transmission elements of $[p^\star]^\dagger S(V^\star,\theta^\star,u^\star)\chi$. Since $y^\star = S(V^\star,\theta^\star,u^\star)\chi$ and net loads pay according to the prices $p^\star$, we observe that $[p^\star]^\dagger y^\star = [p^\star]^\dagger S(V^\star,\theta^\star,u^\star)\chi$. We have proved:

Theorem 1: Under the revenue stream based on the border flow right, the ISO is *exactly* revenue neutral under all dispatch conditions. □

A.7 Payment based on sensitivity to welfare

Assuming appropriate second order conditions [45], the sensitivity of welfare to χ_e is:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \chi_e} &= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star) - [\eta^\star]^\dagger \begin{bmatrix} F_e(V^\star,\theta^\star,u^\star) \\ \mathbf{0} \end{bmatrix}, \\
&= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star) - [\eta_e^\star]^\dagger F_e(V^\star,\theta^\star,u^\star).
\end{aligned}$$

Payment to the owner of element e based on the sensitivity to welfare is given by:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \chi_e}\chi_e &= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e - [\eta_e^\star]^\dagger F_e(V^\star,\theta^\star,u^\star)\chi_e, \\
&= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e - [\eta_e^\star]^\dagger \begin{bmatrix} F_e(V^\star,\theta^\star,u^\star) & \mathbf{0} \end{bmatrix}\chi, \\
&= [p^\star]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e,
\end{aligned}$$

by complementary slackness [52]. That is, we have proved:

Theorem 2: Payment based on sensitivity to welfare is the same as payment based on the border flow right of net injections times nodal prices. □

B. Contingency constraints

B.1 Variables

Following [7], we distinguish contingencies by a superscript ω. We assume that each contingency $\omega \in \Omega$ has an associated probability Π^ω, which may be vanishingly small. The probability of the base-case is $\Pi^0 = 1 - \sum_{\omega\in\Omega}\Pi^\omega$. In each contingency, we consider that the vector of net loads may change from y to $y + \Delta y^\omega$, with associated benefit changed by $\Delta b(y,\Delta y^\omega)$. If only transmission constraints are modelled and there are no losses then we can omit Δy^ω and Δb. Assuming risk neutrality, the objective is now $\Pi^0 b(y) + \sum_\omega \Pi^\omega \Delta b(y,\Delta y^\omega)$.

B.2 Constraints and problem

Under each contingency, there is a specification of Kirchhoff's laws S^ω and constraints F^ω. The solution of the network equations and controls under contingency ω is specified by $V^\omega, \theta^\omega, u^\omega$. The welfare optimization problem becomes:

$$\max_{\substack{y \in \mathbb{R}^{n_y}, \Delta y^\omega \in \mathbb{R}^{n_y}, \forall \omega \in \Omega, \\ V \in \mathbb{R}^{n_V}, V^\omega \in \mathbb{R}^{n_V}, \forall \omega \in \Omega, \\ \theta \in \mathbb{R}^{n_\theta}, \theta^\omega \in \mathbb{R}^{n_\theta}, \forall \omega \in \Omega, \\ u \in \mathbb{R}^{n_u}, u^\omega \in \mathbb{R}^{n_u}, \forall \omega \in \Omega}} \left\{ \Pi^0 b(y) + \sum_{\omega \in \Omega} \Pi^\omega \Delta b(y, \Delta y^\omega) \;\middle|\; \begin{array}{r} y = S(V,\theta,u)\chi, y + \Delta y^\omega = S^\omega(V^\omega, \theta^\omega, u^\omega)\chi, \forall \omega \in \Omega, \\ F(V,\theta,u)\chi \leq \mathbf{0}, F^\omega(V^\omega, \theta^\omega, u^\omega)\chi \leq \mathbf{0}, \forall \omega \in \Omega \end{array} \right\}.$$

We assume that there is a solution $y^\star, \Delta y^{\omega\star}, \forall \omega \in \Omega, V^\star, V^{\omega\star}, \forall \omega \in \Omega, \theta^\star, \theta^{\omega\star}, \forall \omega \in \Omega, u^\star, u^{\omega\star}, \forall \omega \in \Omega$.

B.3 Contingency constraints for example problem

The contingency constraints for the example problem are specified by:

$$\begin{aligned}
\Omega &= \{1,2,3\}, \\
S^\omega &= \begin{bmatrix} S_1^\omega & S_2^\omega & S_3^\omega \end{bmatrix}, \\
\text{where: } \forall \theta^\omega \in \mathbb{R}^2, S_e^\omega(\theta^\omega) &= \begin{bmatrix} -(\theta_k^\omega - \theta_\ell^\omega) & 0 \\ -(\theta_\ell^\omega - \theta_k^\omega) & 0 \end{bmatrix}, \omega = 1,2,3, e \neq \omega, \\
\forall \theta^\omega \in \mathbb{R}^2, S_\omega^\omega(\theta^\omega) &= \mathbf{0}, \omega = 1,2,3, \\
F^1 &= \begin{bmatrix} \mathbf{0} & F_2^1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_3^1 \end{bmatrix}, \\
F^2 &= \begin{bmatrix} F_1^2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_3^2 \end{bmatrix}, \\
F^3 &= \begin{bmatrix} F_1^3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & F_2^3 & \mathbf{0} \end{bmatrix}, \\
\text{where: } \forall \theta^\omega \in \mathbb{R}^2, F_e^\omega(\theta) &= \begin{bmatrix} (\theta_k^\omega - \theta_\ell^\omega) & -1 \end{bmatrix}, \omega = 1,2,3, e \neq \omega.
\end{aligned}$$

The only binding inequality constraints correspond to $\omega = 2,3$ and $e = 1$. Since only transmission contingencies are modelled and since the system is lossless, we can omit Δb and the vectors Δy^ω.

B.4 Lagrangian

The Lagrangian is now:

$$\begin{aligned}
&\mathcal{L}(y, \Delta y^\omega, \omega \in \Omega, V, V^\omega, \omega \in \Omega, \theta, \theta^\omega, \omega \in \Omega, u, u^\omega, \omega \in \Omega, p^0, p^\omega, \omega \in \Omega, \eta^0, \eta^\omega, \omega \in \Omega) \\
&= \Pi^0 b(y) + \sum_{\omega \in \Omega} \Pi^\omega \Delta b(y, \Delta y^\omega) \\
&\quad - [p^0]^\dagger (y - S(V,\theta,u)\chi) - \sum_{\omega \in \Omega} [p^\omega]^\dagger (y + \Delta y^\omega - S^\omega(V^\omega, \theta^\omega, u^\omega)\chi) \\
&\quad - [\eta^0]^\dagger F(V,\theta,u)\chi - \sum_{\omega \in \Omega} [\eta^\omega]^\dagger F^\omega(V^\omega, \theta^\omega, u^\omega)\chi,
\end{aligned}$$

where we denote the dual variables corresponding to the base-case constraints by a superscript 0 to distinguish them from the symbol that we will use for the nodal prices.

Again we assume that there are corresponding Lagrange multipliers $p^{0\star}, p^{\omega\star}, \forall \omega \in \Omega, \eta^{0\star}, \eta^{\omega\star}, \forall \omega \in \Omega$. As in the non-contingency constrained case, we will focus on a particular transmission element and assume that the functions F and S and Lagrange multipliers $\eta^{0\star}$ and $\eta^{\omega\star}$ have been partitioned as before.

B.5 Nodal prices

The nodal prices (for net loads y in the base-case) are now given by $p^\star = p^{0\star} + \sum_{\omega\in\Omega} p^{\omega\star}$. We can interpret these as the sum of prices corresponding to energy and to binding constraints in the base and contingency cases.

If payments to transmission lines are based on the sum over normal and contingency states of the product of net injection or withdrawal in a state times the state dependent LMP then we again have that:

Theorem 3: Under the revenue stream based on the border flow right and contingency flows, the ISO is *exactly* revenue neutral under all dispatch conditions. □

B.6 Payment based on sensitivity to welfare

Again assuming appropriate second order conditions [45], the sensitivity of welfare to χ_e is:

$$\frac{\partial\mathcal{L}}{\partial\chi_e} = [p^{0\star}]^\dagger S_e(V^\star,\theta^\star,u^\star) - \sum_{\omega\in\Omega}[p^{\omega\star}]^\dagger S_e(V^{\omega\star},\theta^{\omega\star},u^{\omega\star}) - [\eta_e^{0\star}]^\dagger F_e(V^\star,\theta^\star,u^\star) - \sum_{\omega\in\Omega}[\eta^{\omega\star}]^\dagger F_e(V^{\omega\star},\theta^{\omega\star},u^{\omega\star}).$$

Again by complementary slackness, payment to the owner of element e based on the sensitivity to welfare is given by:

$$\frac{\partial\mathcal{L}}{\partial\chi_e}\chi_e = \left[[p^{0\star}]^\dagger S_e(V^\star,\theta^\star,u^\star) + \sum_{\omega\in\Omega}[p^{\omega\star}]^\dagger S_e(V^{\omega\star},\theta^{\omega\star},u^{\omega\star})\right]\chi_e. \tag{5}$$

This payment is the sum over base and contingency cases of injections in each case weighted by corresponding nodal prices. We have proved:

Theorem 4: Payment based on sensitivity to welfare is the same as payment based on the border flow right of the sum over normal and contingency states of net injections times nodal prices. □

This result appears to simplify and generalize the analysis in Gribik *et al.* [2]. Similar analysis to the non-contingency constrained case shows that the ISO is revenue neutral.

B.7 Comparison of sensitivity based payment to (1)

The sensitivity based payment (5) is not the same as the expression corresponding to (1) using pre-contingency flows, which in this case would be:

$$[p^{0\star} + \sum_{\omega\in\Omega} p^{\omega\star}]^\dagger S_e(V^\star,\theta^\star,u^\star)\chi_e. \tag{6}$$

The two expressions (5) and (6) differ for element e to the extent that the base-case and contingency flows differ on element e. The payment based on pre-contingency flows (6) is an approximation to the exact expression (5), which provides efficient marginal incentives for transmission expansion.

The pre-contingency and post-contingency flows can be considerably different. For example, for a line e that is the binding contingency for a constraint, the contingency case flow on e is zero, while the base-case flow on e is non-zero. For other lines, however, the base-case and contingency-case flows differ by only a fraction of the base-case flow on the contingency constrained line. It is an empirical question as to whether (5) and (6) differ significantly in practice and we hypothesize that (6) may typically be a useful approximation to (5).