



# Linear Electricity Spot Market Constraints for Managing Post-Separation Frequency Deviations

Stuart Thorncraft, [s.thorncraft@ieee.org](mailto:s.thorncraft@ieee.org)

IEEE PES GM, Tampa Florida, USA, June 24-28 2007

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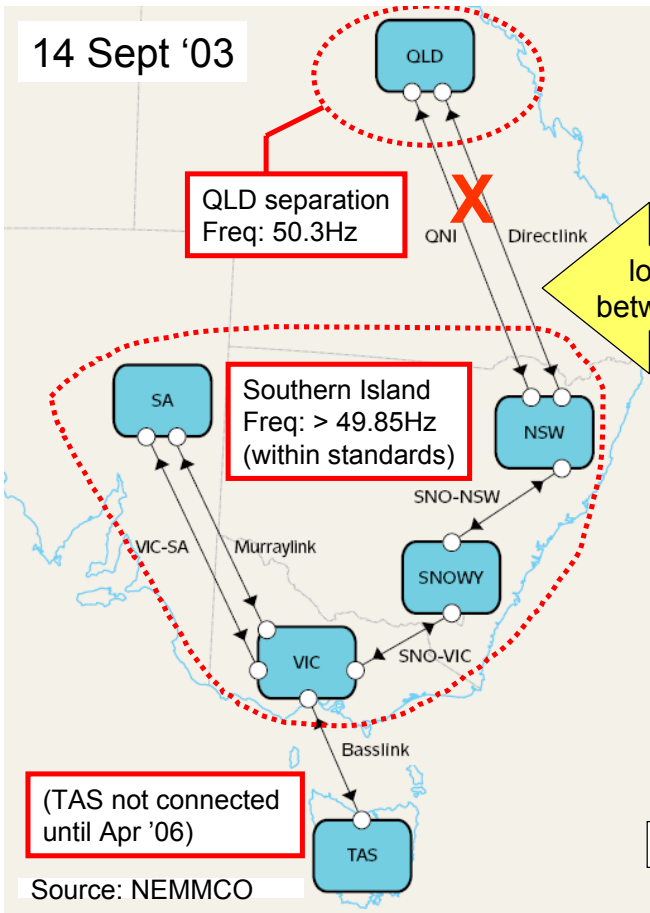
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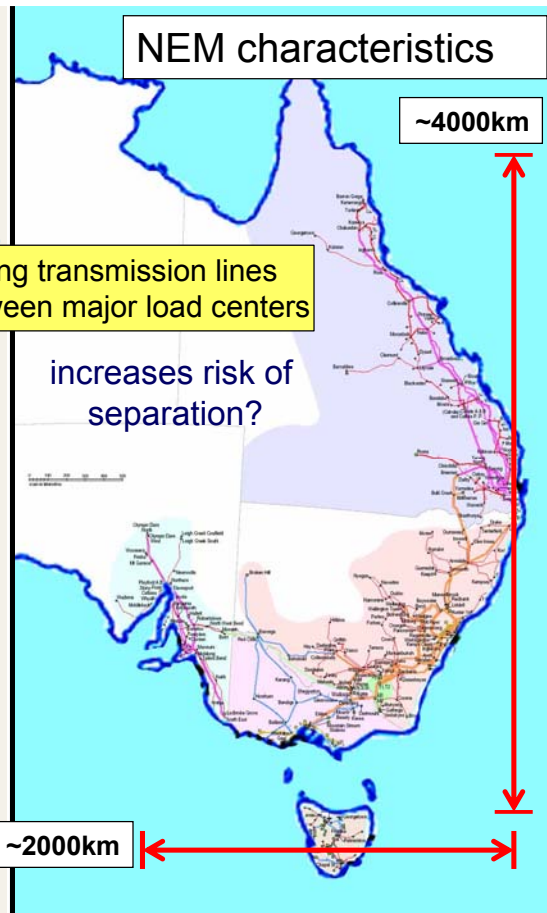
## Outline

- brief motivation for work
  - separation events
  - conceptual view of security management in a restructured electricity industry
- post-separation power system model
- derivation of linear constraint sets from model
- security management & electricity market interface
- illustrative example
- conclusions and further work

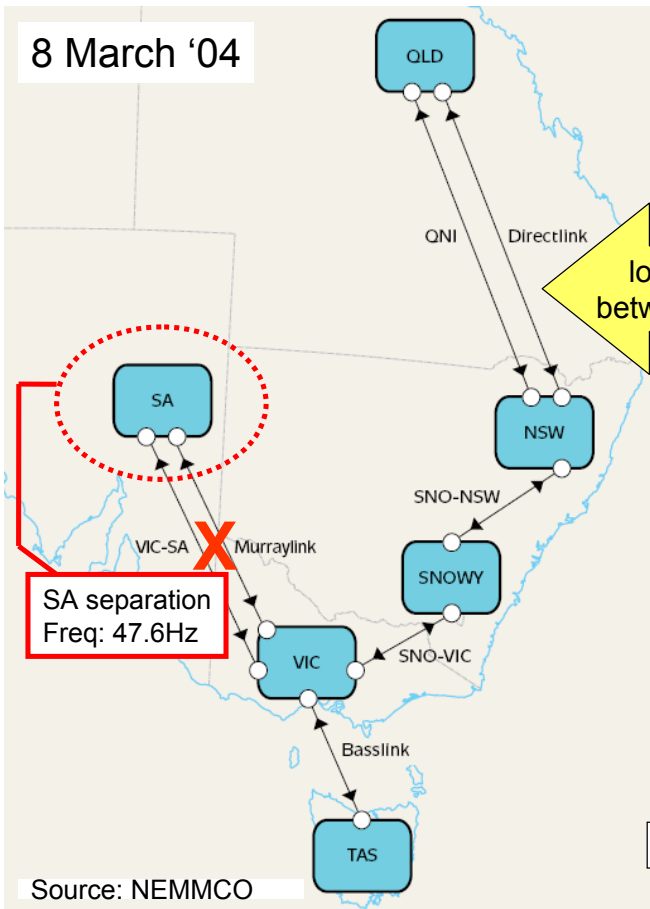
14 Sept '03



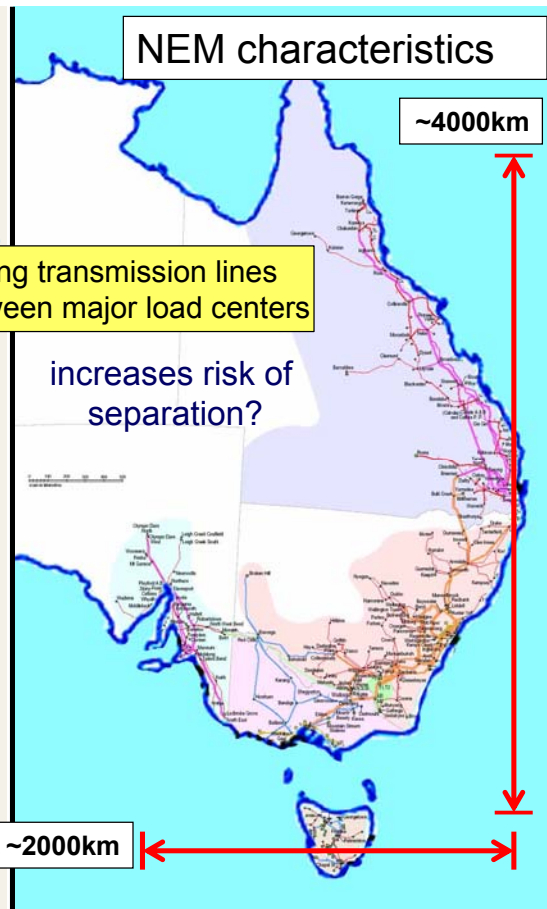
NEM characteristics

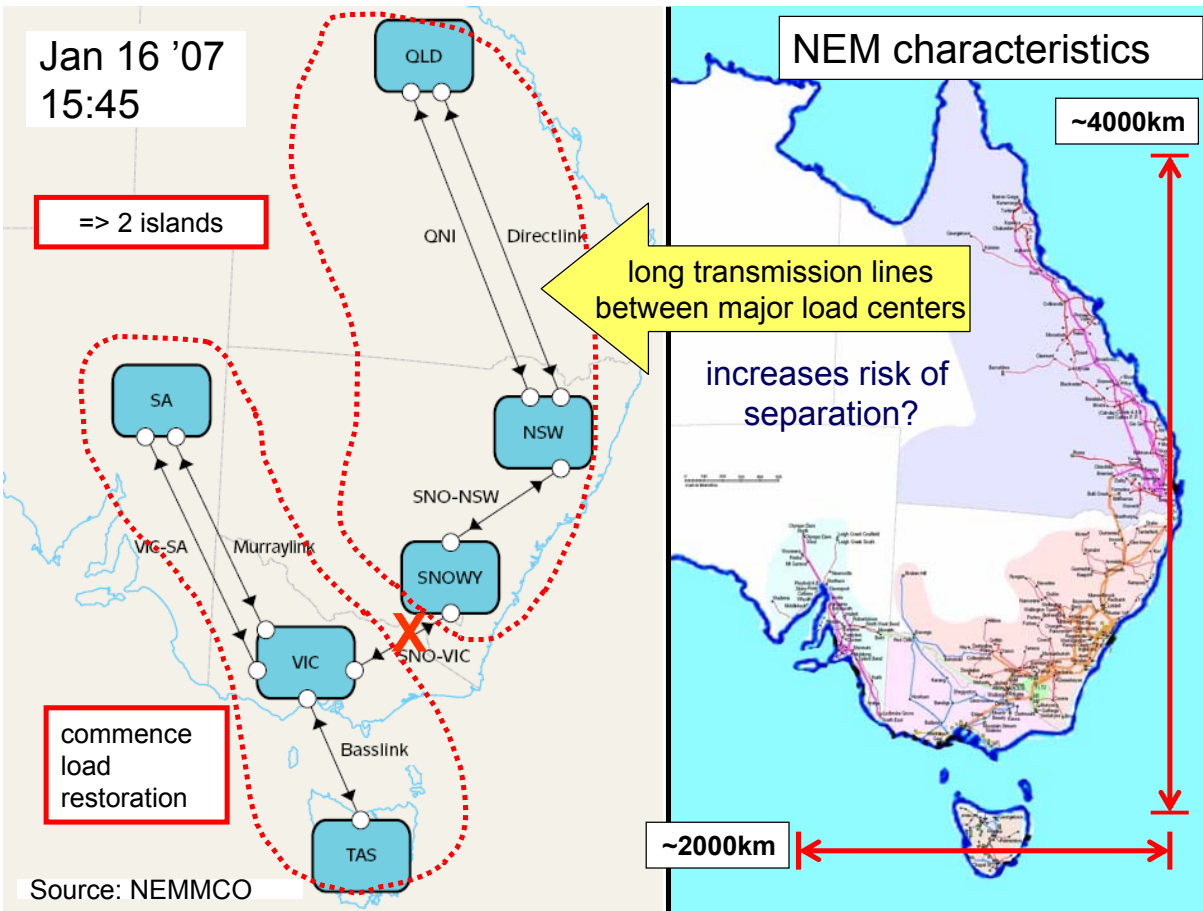
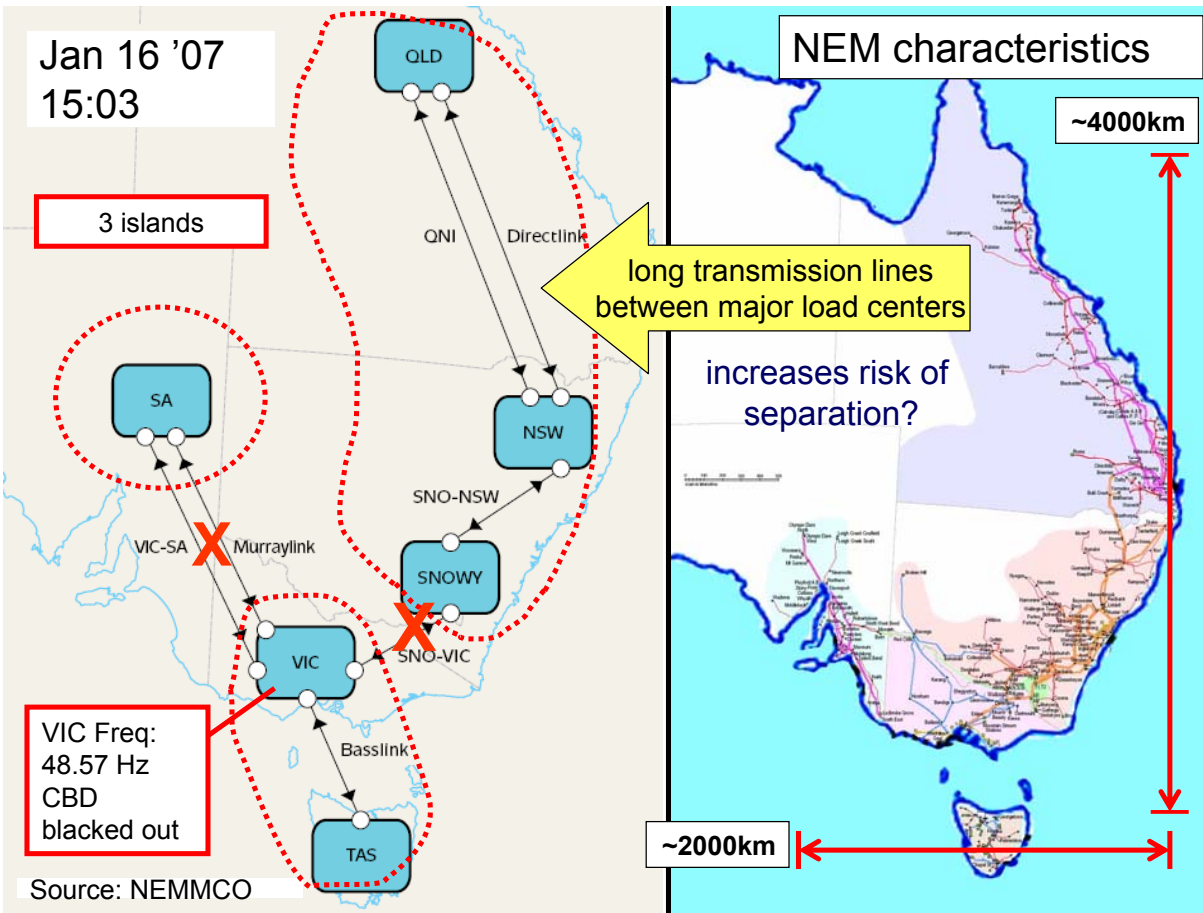


8 March '04



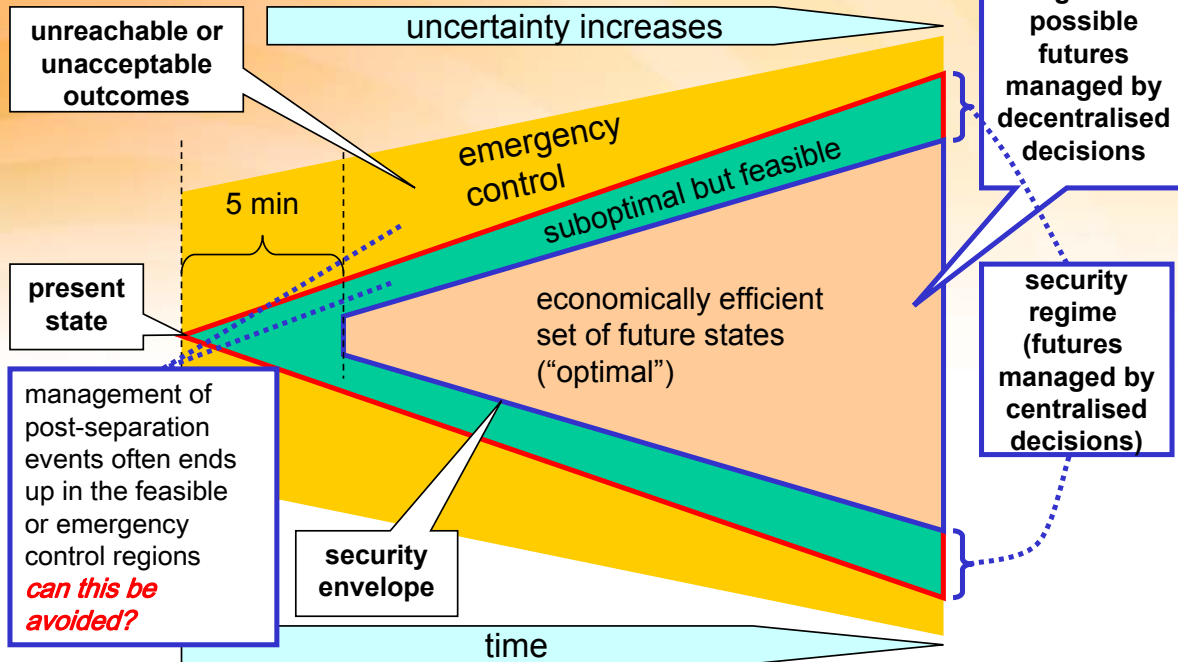
NEM characteristics







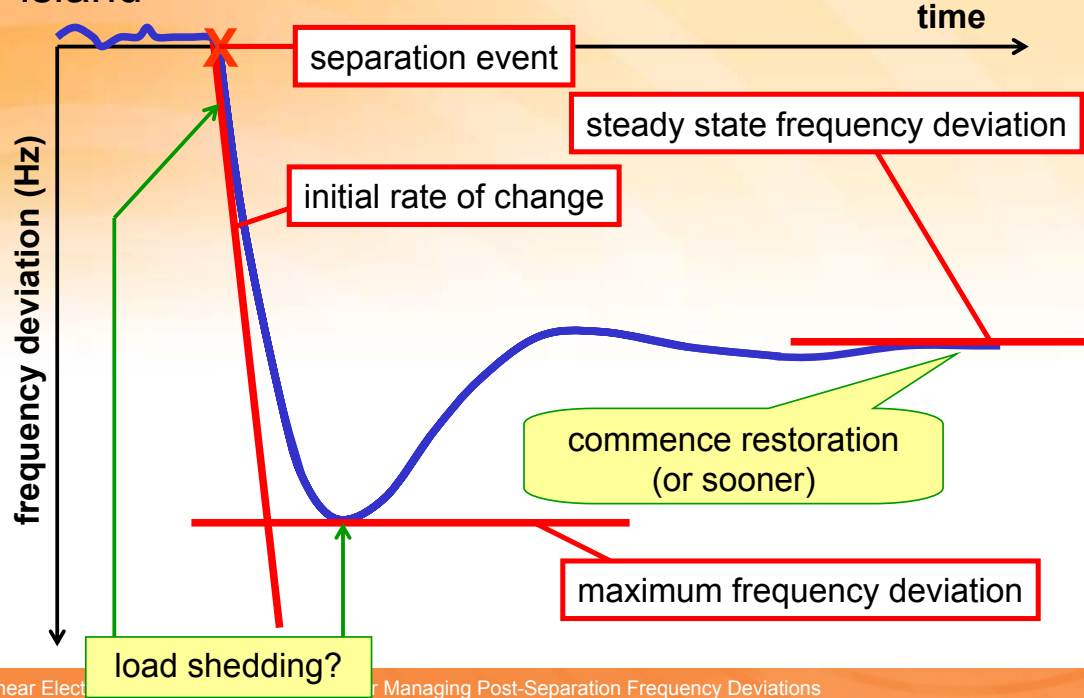
# Security & Commercial Concept



# Security & Market Operation

- conflicting objectives between system operations & market participants
- power system operations concerned with
  - keeping core system intact (has cost implications)
  - Identifying & preventing rare events (potentially 'high impact')
  - uncertain physical outcomes => uncertain control actions
- market & industry participants
  - profit maximisation => push system to boundary
  - commercial transactions can't proceed if system fails
  - consistent & objective decision-making framework reduces uncertainty
- security concept:
  - system operators compute and apply a secure envelope
  - secure envelope is based on objective criteria: system standards
- can the security space for managing post-separation frequency deviations be defined in this way?

## Basic Definitions: Consider an under-frequency island



## Post-Separation Dynamic Power System Model

- low-order frequency response model for each island (deviations):

$$\begin{bmatrix} \dot{f}_i(t) \\ \dot{p}_{i1}(t) \\ \vdots \\ \dot{p}_{iN}(t) \end{bmatrix} = \begin{bmatrix} \delta_i & \sigma_i & \cdots & \sigma_i \\ \gamma_{i1} & \tau_{i1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{iN} & 0 & \cdots & \tau_{iN} \end{bmatrix} \begin{bmatrix} f_i(t) \\ p_{i1}(t) \\ \vdots \\ p_{iN}(t) \end{bmatrix} + \begin{bmatrix} -\sigma_i \\ 0 \\ \vdots \\ 0 \end{bmatrix} p(t), \quad i \in \mathcal{I}$$

$$\delta_i = -\frac{D_i}{2H_i}; \quad \sigma_i = \frac{f_0}{2H_i S_0};$$

$$\gamma_{ij} = -\frac{S_0}{f_0 T_{ij} R_{ij}}; \quad \tau_{ij} = -\frac{1}{T_{ij}}$$

$$\dot{\mathbf{x}}_i(t) = \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{b}_i p(t)$$

$$y_{ik}(t) = \mathbf{e}_{ik}^T \mathbf{x}_i(t)$$

standard LTI system  
for each possible island

## Steady-State Frequency Deviations

- set derivatives to zero & solve

$$\begin{aligned} f_i(\infty) &= y_{i1}(\infty) \\ &= -\mathbf{e}_1^T (A_i)^{-1} \mathbf{b}_i \Delta p \\ &\triangleq K_i \Delta p \\ p_{ij}(\infty) &= y_{i,j+1}(\infty) \\ &= -\mathbf{e}_{j+1}^T (A_i)^{-1} \mathbf{b}_i \Delta p \\ &\triangleq K_{ij} \Delta p \end{aligned}$$

power output of  
generator j,  
island i

power lost or  
gained in island

## Initial Rate of Change of Frequency

- put  $t = 0$  & consider initial conditions

$$\dot{\mathbf{x}}_i(0) = A_i \mathbf{x}_i(0) + \mathbf{b}_i \Delta p$$

initial state (freq &  
power outputs)

$$\begin{aligned} \dot{f}_i(0) &= -\sigma_i \Delta p \\ &\triangleq L_i \Delta p \end{aligned}$$

(not necessary) put  
initial conditions = 0

## Maximum Frequency Deviation (1)

- use standard analytical expression

$$\begin{aligned}\mathbf{x}_i(t) &= \exp(A_i t) \mathbf{x}_i(0) + \int_0^t \exp(A_i(t-\tau)) \mathbf{b}_i d\tau \cdot \Delta p \\ &= A_i^{-1} (\exp(A_i t) - I) \mathbf{b}_i \Delta p\end{aligned}$$

solve for  $t$  to  
find extreme  
values of  
states

$$\begin{aligned}\dot{f}_i(t) &= \mathbf{e}_1^T \dot{\mathbf{x}}_i(t) \\ &= \mathbf{e}_1^T \exp(A_i t) (A_i \mathbf{x}_i(0) + \mathbf{b}_i \Delta p) \\ &= \mathbf{e}_1^T \exp(A_i t) \mathbf{b}_i \Delta p\end{aligned}$$

find  $t = t_{max} \geq 0$   
to make zero

$$\begin{aligned}\dot{p}_{ij}(t) &= \mathbf{e}_{j+1}^T \dot{\mathbf{x}}_i(t) \\ &= \mathbf{e}_{j+1}^T \exp(A_i t) \mathbf{b}_i \Delta p\end{aligned}$$

find  $t = t_j \geq 0$   
to make zero

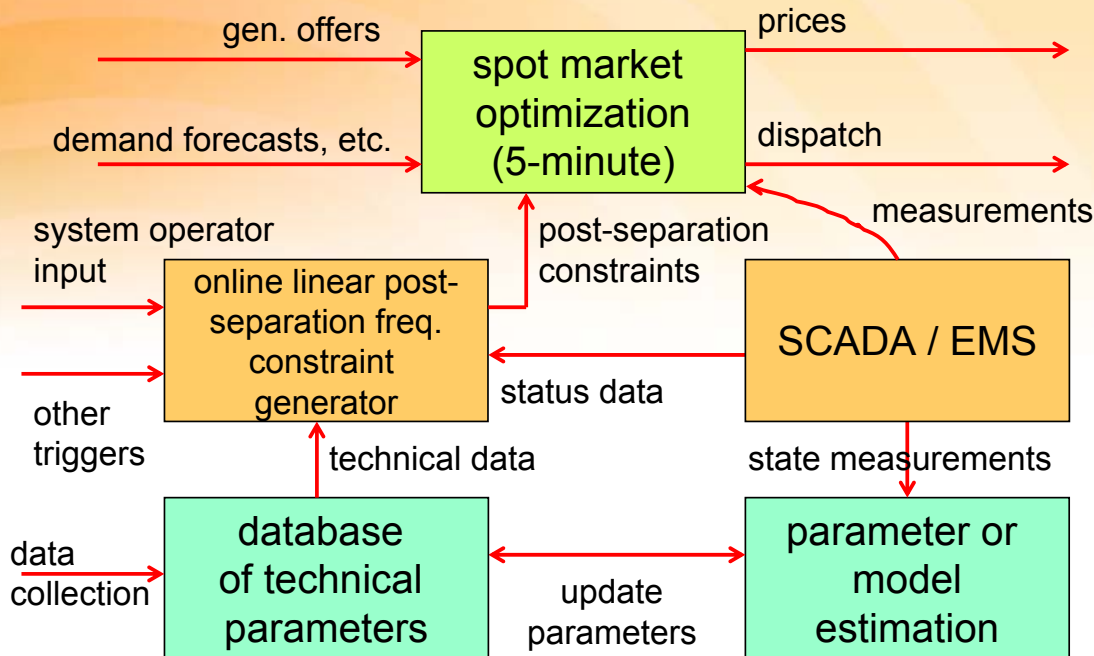
## Maximum Frequency Deviation (2)

- Find  $t_{max}$  &  $t_j$  to get maximum deviations

$$\begin{aligned}f_i(t_{max}) &= \mathbf{e}_1^T A_i^{-1} (\exp(A_i t_{max}) - I) \mathbf{b}_i \Delta p \\ &= M_i \Delta p \\ p_{ij}(t_j) &= \mathbf{e}_{j+1}^T A_i^{-1} (\exp(A_i t_j) - I) \mathbf{b}_i \Delta p \\ &\triangleq \\ &\equiv M_{ij} \Delta p\end{aligned}$$

can also compute maximum  
output deviations

# Automatic Constraint Generation



# Spot Market + Post-Separation Security Constraints LP Optimization (1)

Minimize:

$$J = \sum_{j \in \mathcal{G}} C_j g_j$$

Subject to:

$$0 \leq g_j \leq G_j^{\max}, \quad j \in \mathcal{G}$$

$$|p_k| \leq P_k^{\max}, \quad k \in \mathcal{L}$$

$$\sum_{j \in \mathcal{G}_i} g_j - N_i = \sum_{k \in \mathcal{F}_i} p_k - \sum_{k \in \mathcal{T}_i} p_k, \quad i \in \mathcal{I}$$

basic LP market formulation (lossless model)

$$|f_i(\infty)| \leq F_{ss}^{\max}, \quad i \in \mathcal{I}$$

$$f_i(\infty) = -K_i p_k, \quad i \in \mathcal{I}, \quad k \in \mathcal{T}_i$$

$$f_i(\infty) = K_i p_k, \quad i \in \mathcal{I}, \quad k \in \mathcal{F}_i$$

$$0 \leq g_j - K_{ij} p_k \leq G_j^{\max}, \quad i \in \mathcal{I}, \quad j \in \mathcal{S}_i, \quad k \in \mathcal{T}_i$$

$$0 \leq g_j + K_{ij} p_k \leq G_j^{\max}, \quad i \in \mathcal{I}, \quad j \in \mathcal{S}_i, \quad k \in \mathcal{F}_i$$

post-separation steady-state linear frequency constraints



# Spot Market + Post-Separation Security Constraints LP Optimization (2)

$$|\dot{f}_i(0)| \leq \dot{F}^{\max}, \quad i \in \mathcal{I}$$

$$f_i(0) = -L_i p_k, \quad i \in \mathcal{I}, \quad k \in \mathcal{T}_i$$

$$f_i(0) = L_i p_k, \quad i \in \mathcal{I}, \quad k \in \mathcal{F}_i$$

post-separation rate-of-change of frequency linear constraints

$$|f_i(t^{\max})| \leq F^{\max}, \quad i \in \mathcal{I}$$

$$f_i(t^{\max}) = -M_i p_k, \quad i \in \mathcal{I}, \quad k \in \mathcal{T}_i$$

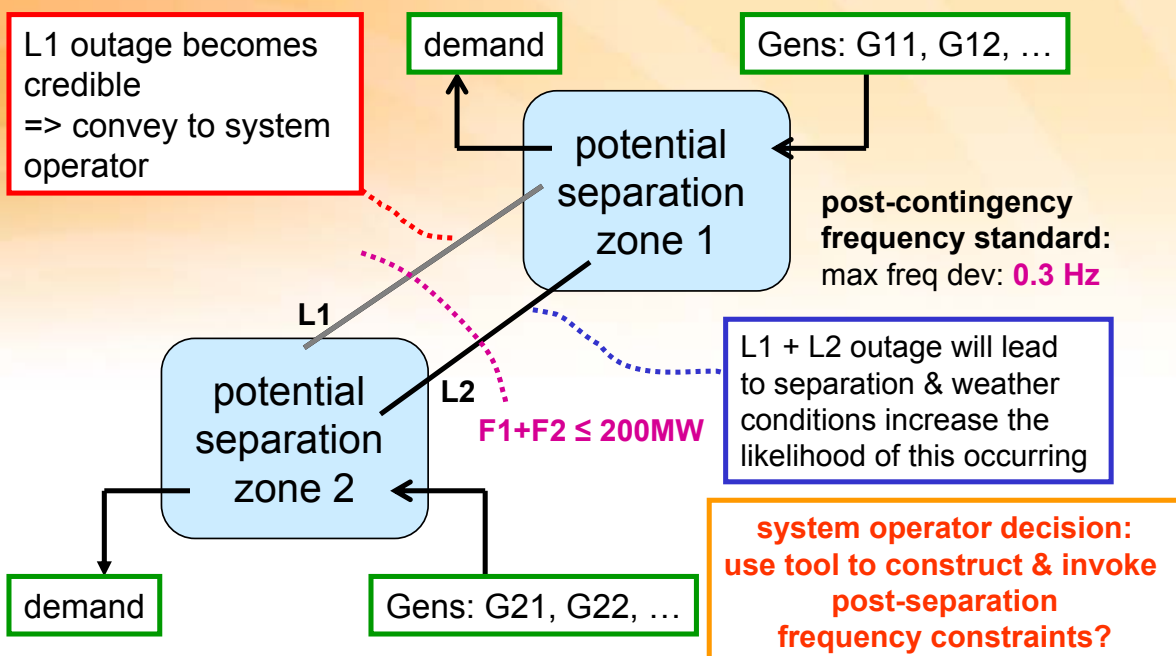
$$f_i(t^{\max}) = M_i p_k, \quad i \in \mathcal{I}, \quad k \in \mathcal{F}_i$$

$$0 \leq g_j - M_{ij} p_k \leq G_j^{\max}, \quad i \in \mathcal{I}, \quad j \in \mathcal{S}_i, \quad k \in \mathcal{T}_i$$

$$0 \leq g_j + M_{ij} p_k \leq G_j^{\max}, \quad i \in \mathcal{I}, \quad j \in \mathcal{S}_i, \quad k \in \mathcal{F}_i$$

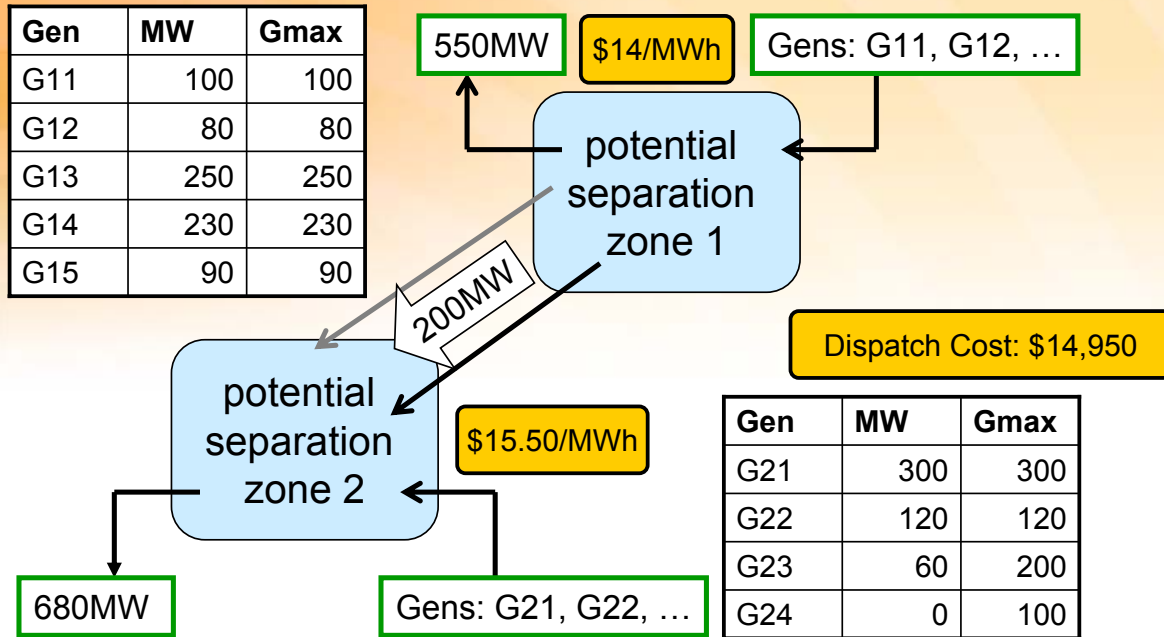
post-separation frequency deviation linear constraints

## Simple example: hypothetical scenario

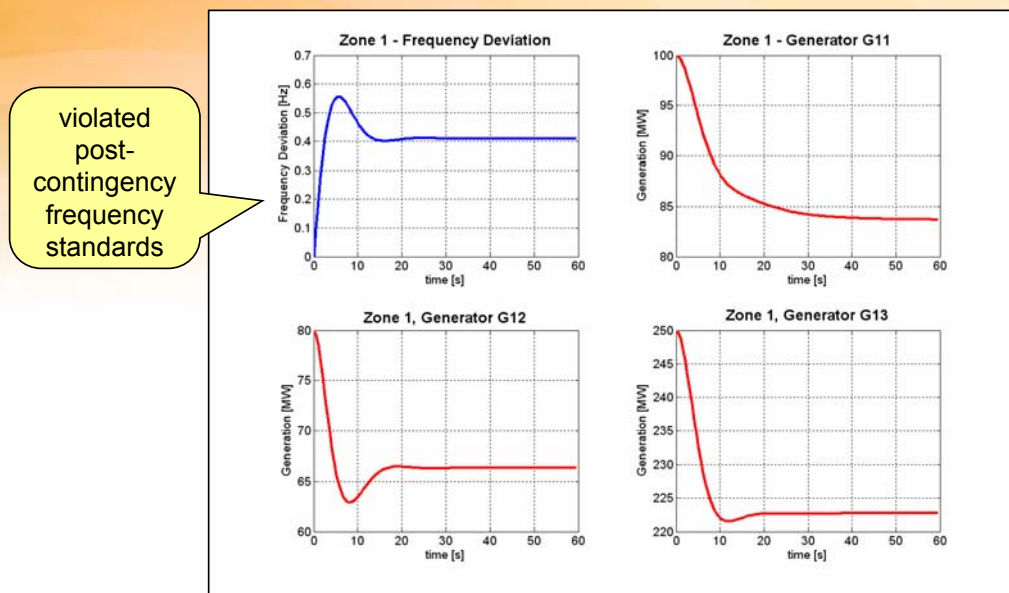




# Simple example: without PSF constraints

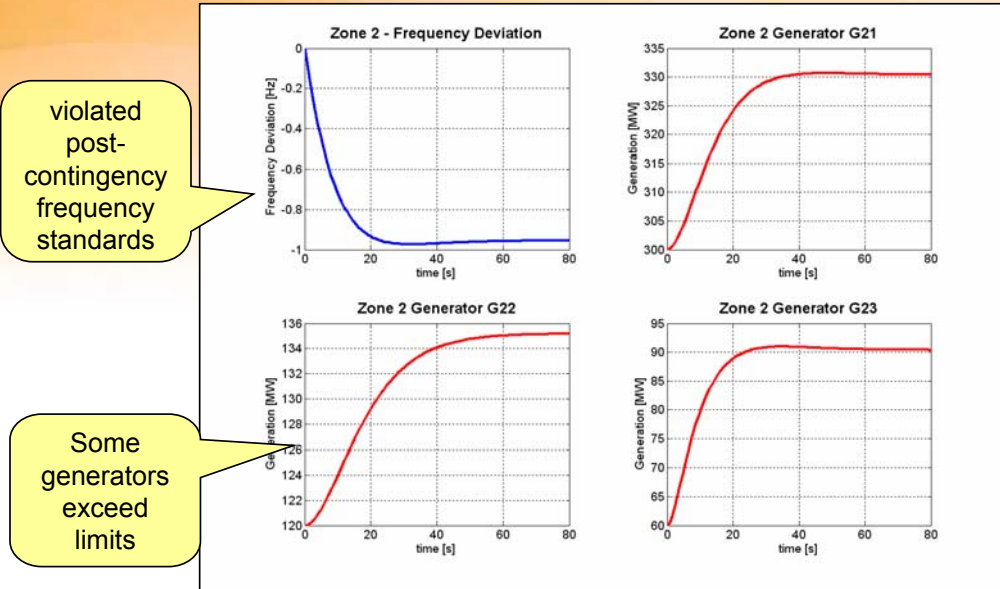


# Simple example: post-separation outcome in zone 1 without PSF constraints

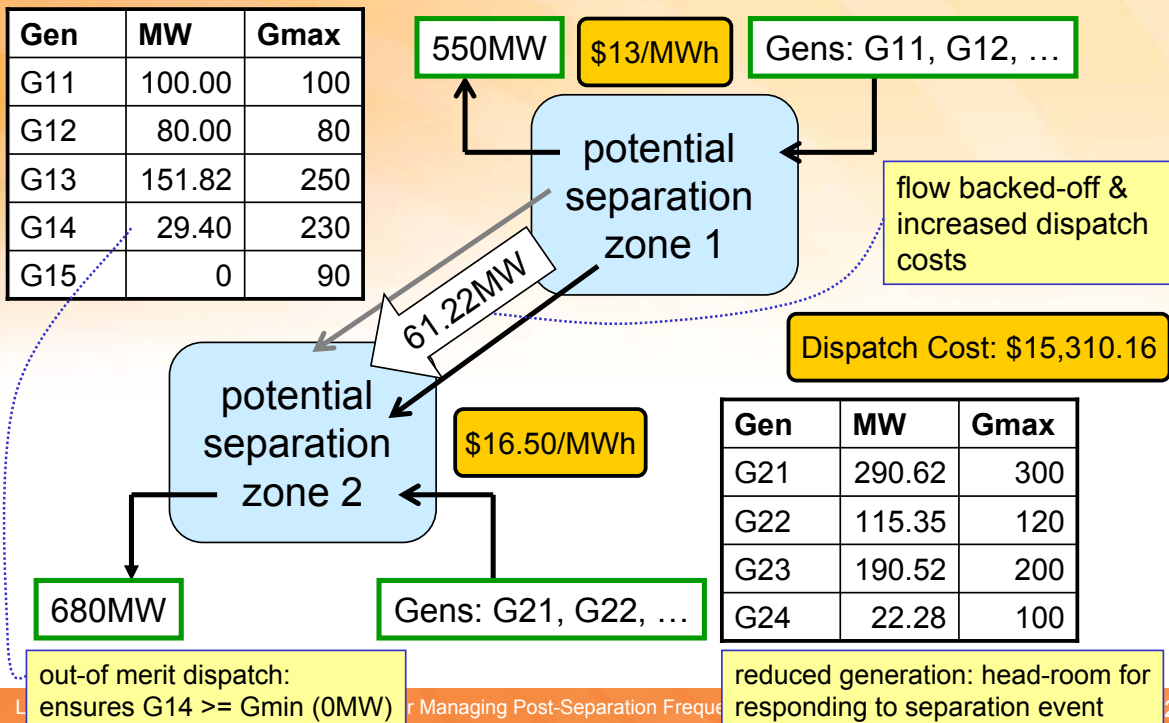




# Simple example: post-separation outcome in zone 2 without PSF constraints



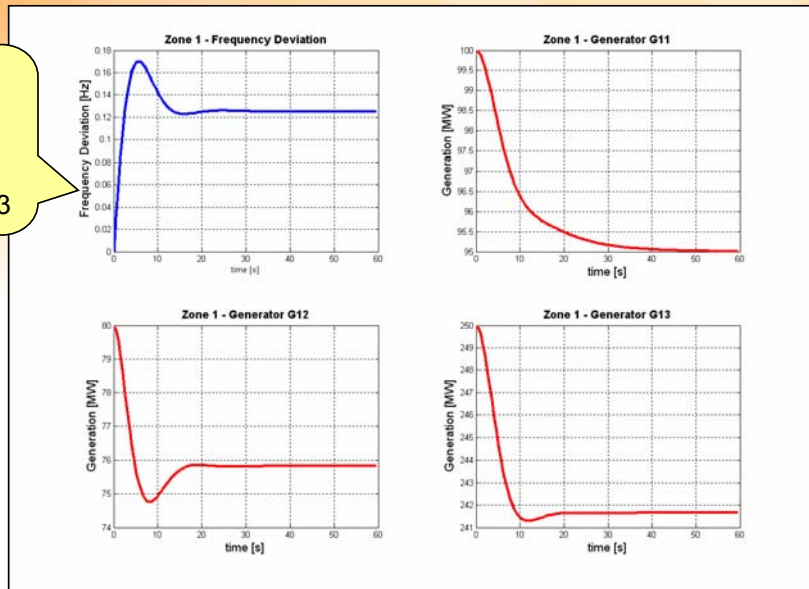
# Simple example: with PSF constraints





# Simple example: post-separation outcome in zone 1 with PSF constraints

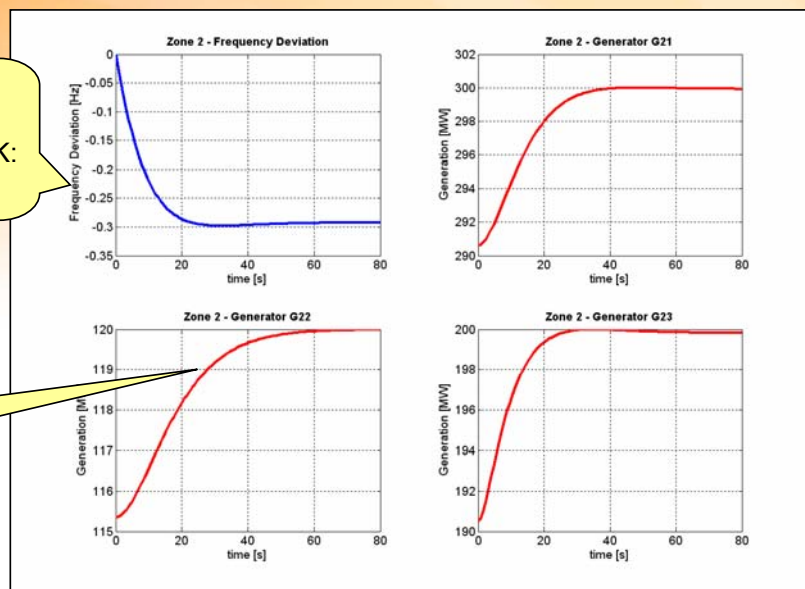
post-contingency frequency is OK: freq dev < 0.3



# Simple example: post-separation outcome in zone 2 with PSF constraints

post-contingency frequency is OK: freq dev ≤ 0.3

generators within limits





# Variation – change in freq standards

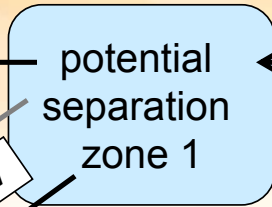
- say the post-contingency frequency standard is modified to be:
  - maximum post-contingency frequency excursion  $\leq 0.3$  Hz; and
  - steady-state frequency deviations within 0.2 Hz
- Invoke 2 sets of post-separation frequency constraints:
  - one set to ensure max frequency deviation is  $\leq 0.3$  Hz; and
  - one set to ensure steady-state frequency deviations within 0.2 Hz



# Simple example: **with** alternative PSF constraints

Gen	MW	Gmax
G11	100.00	100
G12	80.00	80
G13	141.66	250
G14	20.01	230
G15	0	90

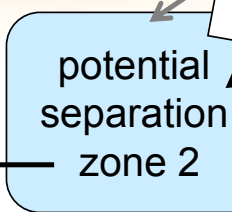
550MW    \$13/MWh    Gens: G11, G12, ...



flow backed-off even more & dispatch costs a bit higher

41.67MW

Dispatch Cost: \$15,345.25



\$16.50/MWh

Gen	MW	Gmax
G21	293.62	300
G22	116.84	120
G23	193.55	200
G24	34.33	100

Gens: G21, G22, ...

680MW

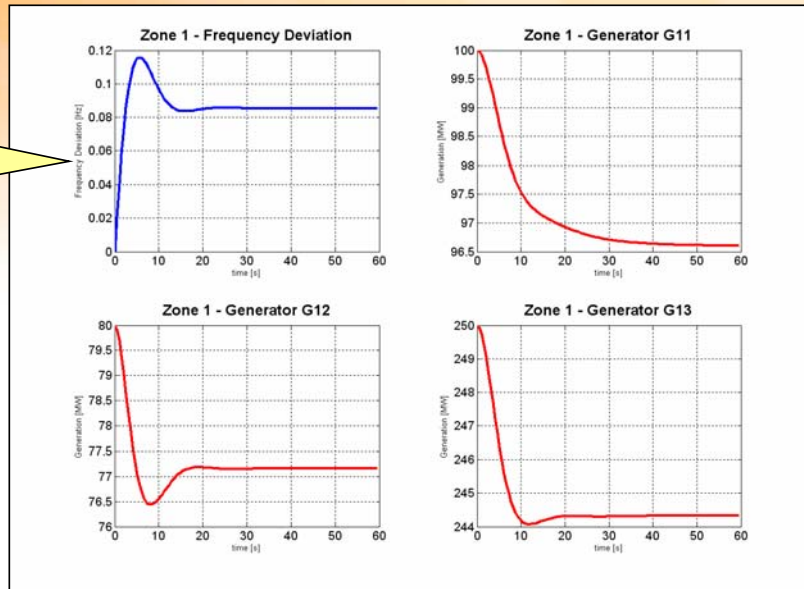
slight variation to dispatch pattern

similar to previously – gens backed off (not as much)



# Simple example: post-separation outcome in zone 1 with alternative PSF constraints

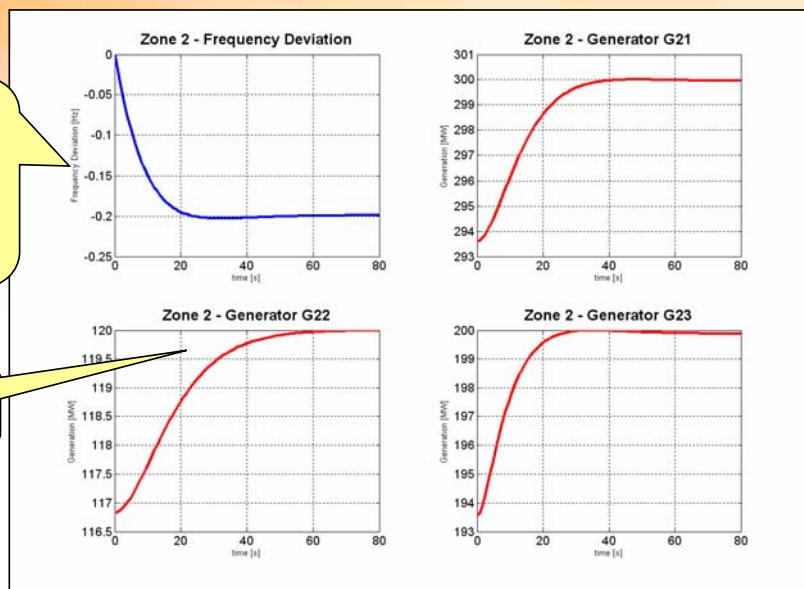
post-contingency frequency OK



# Simple example: post-separation outcome in zone 2 with alternative PSF constraints

post-contingency frequency is OK – deviations hit -0.2Hz

generators within limits





## Conclusions

- separation events are rare but are high impact therefore warrants investigation into mitigation
- important to have a consistent & well-defined interface between security processes & electricity market that:
  - enables system operators to protect the core system
  - can still enable market to proceed
- shown a simple way of linking the following:
  - dynamic power system model
  - post-contingency frequency standards
  - security envelope & system operator decision-making
  - interface to electricity market
- while the process may only be used infrequently, it could prevent high-costs of a post-separation frequency collapse
- more research to be done though!



S. R. Thorncraft, H. R. Outhred, D. J. Clements, "Linear Electricity Spot Market Constraints for Managing Post-Separation Frequency Deviations", to be published in the Proceedings of the IEEE PES General Meeting Tampa Florida, USA, June 24-28 2007, available: [www.ceem.unsw.edu.au](http://www.ceem.unsw.edu.au).

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